1. Parametrize implicitly given curves below:

(a)
$$x^2 - y = 0$$

(b) $x = f(y)$, where $f : \mathbb{R} \to \mathbb{R}$ is a continuous
(c) $(x - p)^2 + (y - q)^2 = r^2$ for fixed $p, q, r \in \mathbb{R}$,

(d)
$$\frac{x^2}{a^2} + \frac{y}{b^2} = 1$$

2. Parametrizations

$$\mathbf{p}_1(t) = (\cos t, \sin t, t)$$

in
$$\mathbf{p}_2(t) = (\sin t, \cos t, t)$$

function,

determine two *helices* in \mathbb{R}^3 .

Find the points of intersection of these two curves. What is the angle of intersection at these points?

- 3. The curve *K* is parametrized by $\mathbf{r}(t) = [x(t), y(t)]^{\mathsf{T}} = [t^3 4t, t^2 4]^{\mathsf{T}}$.
 - (a) Find the intersections of the curve with the coordinate axes *x* and *y*.
 - (b) Write down the equation of the tangent to K at t = 1.
 - (c) Find the points where the tangents are parallel to the coordinate axes.
 - (d) Is there a point of self-intersection on *K*?
 - (e) Sketch the curve *K*.
- 4. Find a parametrization of the epicycloid (and the hypocycloid), i.e. the curve formed by tracing a point on the circumference of a circle with radius *r* which rolls around (inside) a fixed circle of radius *R*.
- 5. Intersection points of planar polygonal chains. Write an octave function P = presecisce(A, B) that determines whether two line segments $\overline{A_1A_2}$ and $\overline{B_1B_2}$ intersect and if so returns the intersection. The line segments are represented by matrices

$$A = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} \text{ in } B = \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}.$$

<u>Two</u> polygonal chains *K* and *L* are defined by the sequence of line segments $\overline{A_1A_2}, \overline{A_2A_3}, \dots$ and $\overline{B_1B_2}, \overline{B_2B_3}, \dots$ Write a function P = presecisca(A, B) that returns all the intersections of the polygonal chains *K* and *L*. The points that define the curves *K* and *L* and the return value *P* are represented by matrices:

$$A = \begin{bmatrix} x_1 & x_2 & \cdots & x_k \\ y_1 & y_2 & \cdots & y_k \end{bmatrix}, B = \begin{bmatrix} u_1 & u_2 & \cdots & u_\ell \\ v_1 & v_2 & \cdots & v_\ell \end{bmatrix} \text{ and } P = \begin{bmatrix} s_1 & s_2 & \cdots & s_m \\ t_1 & t_2 & \cdots & t_m \end{bmatrix}.$$

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6. Intersection points of two parametrically given curves. We are given parametrizations of two curves, *K* and *L*, in the plane \mathbb{R}^2 . Our task is to find all the points of intersection of *K* and *L*.

Let $\mathbf{p}(t)$ and $\mathbf{q}(t)$ be the parametrizations of *K* and *L* defined on intervals I = [a, b] and J = [c, d] respectively.

Find the points of intersection using the following procedure:

- (a) Divide the intervals *I* and *J* into subintervals of length h > 0, where *h* is sufficiently small.
- (b) Approximate *K* and *L* with polygonal chains determinned by evaluating the parametrizations at subdivision points of *I* and *J* and find the intersections of the polygonal chains.
- (c) Use the points of intersection of the polygonal approximations as an initial guess for Newton's iteration, which will determine the actual intersection points of *K* and *L* (much) more accurately.

You will also need the derivatives of the parametrisations $\dot{\mathbf{p}}$ and $\dot{\mathbf{q}}$ in order to use Newton's iteration. Write the function P = presekKrivulj(p, pdot, intp, q, qdot, intq, h) which returns the intersections of two parametrized curves.