VAJE 4

24.(1) $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$A^\dagger=\begin{pmatrix} a & c \\ b & d \end{pmatrix}^* = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}$$

$$(A^\dagger)^\dagger = \begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix}^* = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \qquad |A| = ad-bc$$

$$(A^{-1})^\dagger = \frac{1}{a^*d^*-b^*c^*}\begin{pmatrix} d^* & -c^* \\ -b^* & a^* \end{pmatrix}$$

$$(A^\dagger)^{-1} = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}^{-1} = \frac{1}{a^*d^*-b^*c^*}\begin{pmatrix} d^* & -c^* \\ -b^* & a^* \end{pmatrix}$$

(2) $$(A+B)^\dagger = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}^\dagger \quad \dots$$

(3) $$(AB)^\dagger = \left[\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\right]^\dagger = \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{pmatrix}^\dagger \text{ etc} \dots$$

$$= \begin{pmatrix} a_{11}^*b_{11}^*+a_{12}^*b_{21}^* & a_{21}^*b_{11}^*+a_{22}^*b_{21}^* \\ & \dots \end{pmatrix}$$

$$B^\dagger A^\dagger = \begin{pmatrix} b_{11}^* & b_{21}^* \\ b_{12}^* & b_{22}^* \end{pmatrix}\begin{pmatrix} a_{11}^* & a_{21}^* \\ a_{12}^* & a_{22}^* \end{pmatrix} = \begin{pmatrix} a_{11}^*b_{11}^*+a_{12}^*b_{21}^* & b_{11}^*a_{21}^*+b_{21}^*a_{22}^* \\ & \dots \end{pmatrix}$$

Kronecker

$$A\otimes B = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & \\ a_{11}b_{21} & a_{11}b_{22} & \\ & & \dots \end{pmatrix}$$

(4) $c = 1e_1 + 2e_2 \qquad \varepsilon_1 = \frac{1}{\sqrt2}e_1 + \frac{1}{\sqrt2}e_2 \qquad \varepsilon_2 = \frac{1}{\sqrt2}e_1 - \frac{1}{\sqrt2}e_2$

$e_1 = \frac{1}{\sqrt2}\varepsilon_1 + \frac{1}{\sqrt2}\varepsilon_2 \qquad e_2 = \frac{1}{\sqrt2}\varepsilon_1 - \frac{1}{\sqrt2}\varepsilon_2$

$$c = \frac{1}{\sqrt2}\varepsilon_1 + \frac{1}{\sqrt2}\varepsilon_1 + 2\left(\frac{1}{\sqrt2}\varepsilon_1 - \frac{1}{\sqrt2}\varepsilon_2\right) = \frac{3}{\sqrt2}\varepsilon_1 - \frac{1}{\sqrt2}\varepsilon_2$$

$\{A,B\} = AB + BA$

25. 1) $[A,B] = AB - BA$

$[B,A] = BA - AB = -[A,B]$

2) $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

σ_1 σ_2 σ_3

3) $[\sigma_x, \sigma_y] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = 2i\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$[\sigma_i, \sigma_j] = 2i\varepsilon_{ijk}\sigma_k$ $\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$ $\varepsilon_{ijk} = \begin{cases} 1 & (123), (231), (312) \\ -1 & (321), (132), (213) \\ 0 & \text{sicer} \end{cases}$

4) $\sigma_i^2 = \mathbb{1}$

5) $-i\sigma_1\sigma_2\sigma_3 = -i\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -i\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbb{1}$

6) $\left.\begin{array}{l} \det \sigma_i = -1 \\ \operatorname{tr} \sigma_i = 0 \end{array}\right\}$ $\lambda_{1,2} = \pm 1$

7) $\sigma_i^+ = \sigma_i$

8) $\{\sigma_i, \sigma_j\} = 2\delta_{ij}\mathbb{1}$ $\sigma_1\sigma_2 + \sigma_2\sigma_1 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = 0$

9) $\sigma_i\sigma_j = \delta_{ij}\mathbb{1} + i\varepsilon_{ijk}\sigma_k$

26. $[A,B]^+ = (AB - BA)^+ = B^+A^+ - A^+B^+ = [B^+, A^+]$

$C = i[A,B]$

$C^+ = -i[A,B]^+ = -i[B^+, A^+] = -i[B,A] = i[A,B] = C$

Za poljubno A: $C = A + A^+$ je Hermitska. $C = A - A^+$ je anti-Hermitska.

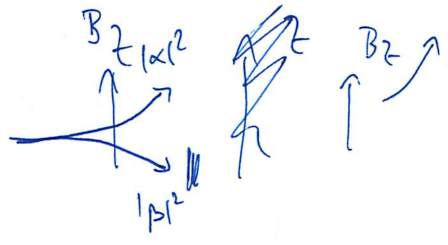
Za poljubno M: $M = A + B$, $A^+ = A$ in $B^+ = -B$, z $A = \frac{1}{2}(M + M^+)$

$B = \frac{1}{2}(M - M^+)$

$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$

33)

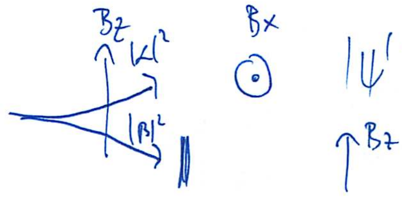
a)



$|\alpha|^2$

$|\psi'\rangle = |\uparrow\rangle$

b)

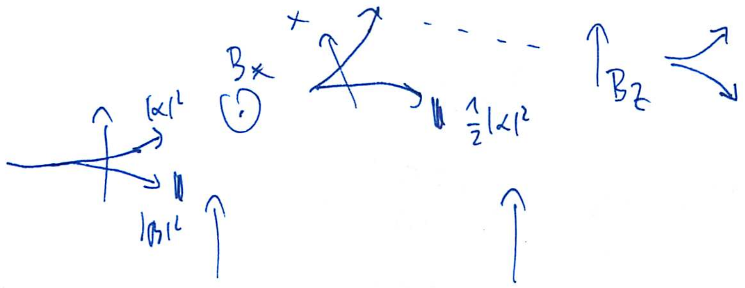


$|\psi'\rangle = |\uparrow\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle$

B_z

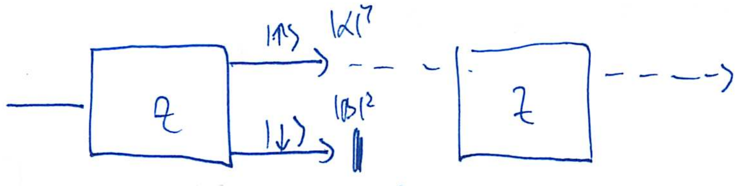
$|\alpha|^2/4$

c)



$|\psi'\rangle = |\uparrow\rangle$ $\quad$ $|\psi'\rangle = |+\rangle$

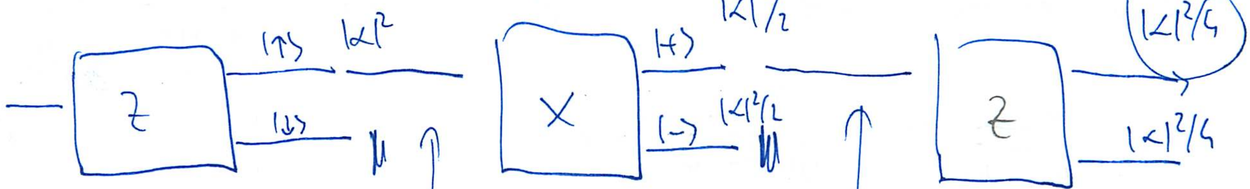
a)



$|\alpha|^2$

$|\psi'\rangle = |\uparrow\rangle$

b)



$|\uparrow\rangle$

$|\psi'\rangle = |\uparrow\rangle$ $\quad$ $|\psi'\rangle = |+\rangle$

29) a) $\sigma_x, \sigma_y, \sigma_z$ $\det\begin{pmatrix}-\lambda & 1\\ 1 & -\lambda\end{pmatrix} = \lambda^2 - 1 = 0$ $\lambda = \pm 1$ $\lambda = 1$ $\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$

etc. Zapis. $|+z\rangle, |-z\rangle \Longleftrightarrow |\uparrow\rangle, |\downarrow\rangle$ $|+x\rangle, |-x\rangle \Longrightarrow |+\rangle, |-\rangle$ $a = b = 1/\sqrt{2}$

b) $\begin{cases} |\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix} \; |\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix} \\ |+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \; |-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\end{cases}$

c) $|+\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$ $|-\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\rangle$

zveze! ↳ in obratno $|\uparrow\rangle = 1/\sqrt{2}\,|+\rangle + 1/\sqrt{2}\,|-\rangle$

operator vs. matrična rep.

27) $A = \cancel{\sum_{\lambda_i} \lambda_i |i\rangle\langle i|}$ pič baza lastnična $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$.

$|1\rangle$ $|+x\rangle = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}$ $|2\rangle$ $|-x\rangle = \begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix}$

$$A = \sum_{ij} |i\rangle A_{ij} \langle j| \qquad A_{ij} = \langle i|A|j\rangle$$

$$A_{11} = (1/\sqrt{2} \;\; 1/\sqrt{2})\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} = (1/\sqrt{2} \;\; 1/\sqrt{2})\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix} = 0$$

$$A_{12} = (1/\sqrt{2} \;\; 1/\sqrt{2})\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix} = (1/\sqrt{2} \;\; 1/\sqrt{2})\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} = 1 = A_{21}^*$$

$$A_{22} = (+ \;\; -)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}+\\ -\end{pmatrix} = (+ \;\; -)\begin{pmatrix}+\\ +\end{pmatrix} = 0$$

$$A_{ij} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

30) $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ Kolokvij: $\alpha = 2$ $\beta = 1 + \sqrt{3}\,i$ → $1/2$

a) α, β poljubna $\langle\psi|\psi\rangle = |\alpha|^2 + |\beta|^2$ $|\varphi\rangle = \frac{1}{\sqrt{\langle\psi|\psi\rangle}}|\psi\rangle$

$$|\varphi\rangle = \frac{\alpha}{\sqrt{|\alpha|^2+|\beta|^2}}|\uparrow\rangle + \frac{\beta}{\sqrt{|\alpha|^2+|\beta|^2}}|\downarrow\rangle$$

b) $p_\uparrow = \frac{|\alpha|^2}{|\alpha|^2+|\beta|^2}$ $p_\downarrow = \frac{|\beta|^2}{|\alpha|^2+|\beta|^2}$ $\sum = 1$

c) $|\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix} = \gamma\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} + \delta\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix}$ → $\gamma = 1/\sqrt{2}$ $\delta = 1/\sqrt{2}$

50:50

d) Globalna faza ni merljiva. $|\psi_1\rangle$ in $|\psi_2\rangle = e^{i\phi}|\psi_1\rangle$

Relativna pa je: $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\varphi}|1\rangle)$ → $\cos^2\varphi/2$ da je v $|+\rangle$.

σ_x $1/\sqrt{2}(|0\rangle + |1\rangle)$