Name and surname

Osnove matematične analize: računski izpit, izredni rok

2. julij 2021

Student ID

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Time limit is 60 minutes. You may use 2 A4-sized sheets of paper with formulas. The use of electronic devices (calculator, phone) is prohibited. Justify all your answers!

Write each problem onto a separate sheet of paper. If you are writing onto a blank piece of paper rather than the problem sheet, please sign your name at the top of every page, write the problem number at the top as well and scan the problems in the correct order. Thanks!

Question 1 (30 marks)

a) (14 marks) Find all solutions $z \in \mathbb{C}$ of the equation

$$z^3 + 8i = 0$$

b) (16 marks) Find all solutions $z \in \mathbb{C}$ of the equation

$$z^3\bar{z} - z^4 + iz^3 + 8i\bar{z} - 8iz - 8 = 0$$

Hint: Decompose the expression and find numbers a, b and c such that

$$(z^3 + 8i)(az + b\bar{z} + c) = z^3\bar{z} - z^4 + iz^3 + 8i\bar{z} - 8iz - 8$$

Question 2 (35 marks)

a) (10 marks) Calculate the limit $\lim_{x\to 0} x^2 \log(x^2)$.

b) (15 marks) Calculate the indefinite integral $\int x^2 \log(x^2)\, dx$ using per partes.

c) (10 marks) Calculate the definite integral $\int_{-1}^{1} x^2 \log(x^2)\, dx$.

Question 3 (30 marks)

Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = x^3 + y^3 - xy.$$

a) (20 marks) Find and classify all stationary points of the function f.

b) (10 marks) Find the constrainted extrema of the function $g(x, y) = y$, subjected to the constraint $x^3 + y^3 = 1$.