

Osnove matematične analize: 1. računski izpit

19. januar 2021

Time limit is 60 minutes. You may use 2 A4-sized sheets of paper with formulas. The use of electronic devices (calculator, phone) is prohibited. Justify all your answers!

Write each problem onto a separate sheet of paper. If you are writing onto a blank piece of paper rather than the problem sheet, please sign your name at the top of every page, write the problem number at the top as well and scan the problems in the correct order. Thanks!

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Question 1 (30 marks)

For all $n \in \mathbb{N}$ we can write $z_n = (1 + \sqrt{2}i)^n$ in the form $a_n + b_n\sqrt{2}i$ for some uniquely determined $a_n, b_n \in \mathbb{Z}$. So, $a_n = \operatorname{Re}(z_n)$ and $b_n = \operatorname{Im}(z_n)/\sqrt{2}$.

a) (5 marks) Find a_0, b_0, a_1, b_1, a_2 and b_2 .

b) (10 marks) Express a_{n+1} and b_{n+1} using a_n and b_n . (Hint: Express z_{n+1} using z_n .)

c) (5 marks) Compute $\operatorname{Im}(z_4)$. (You may use the previous point or compute it directly.)

d) (10 marks) From the recursions for a_{n+1} and b_{n+1} we can derive the recursive expression $a_{n+2} = 2a_{n+1} - 3a_n$. Using this expression compute $\operatorname{Re}(z_7)$.

Question 2 (30 marks)

Question 3 (35 marks)

Let

$$f(x) = \frac{x}{\sqrt{x^2 - 1}}$$

a) (18 marks) Compute the indefinite integrals $\int f(x)dx$ and $\int f(x)^2 dx$.

b) (10 marks) Does any of the two improper integrals

$$\int_1^2 f(x)dx \quad \text{or} \quad \int_1^2 f(x)^2 dx$$

exist? If so, compute it. Explain your answers!

c) (7 marks) Find the volume of the solid of rotation obtained by rotating the graph of the function $f(x)$ around the x -axis over the interval $x \in [2, 3]$.

If the results contain roots, logarithms, etc. you can leave them as is.