3. popravni kolokvij iz Osnov matematične analize

(Ljubljana, 1. september 2017)

Time limit: 90 minutes. Every exercise is worth the same amount of points. Read the instructions carefully. You may use two A4 sheets with formulas. The results will be published on `ucilnica.fri.uni-lj.si`.

Justify all your answers!

1. (a) Using the polar coordinates and De Moivre's formula calculate $(-\sqrt{3}+3i)^7$.
   (b) Find a complex number z which is a solution of the equation $|z|+z=2+i$.

2. A sequence $(a_n)_{n\in\mathbb{N}}$ is given by the initial term $a_0=0$ and the recursive formula
$$a_{n+1}=\frac{3a_n+2}{5}\,.$$
   (a) Find a_1 and a_2.
   (b) Find all candidates for the limit of the sequence a_n.
   (c) Use mathematical induction to first show that the sequence a_n is bounded from above by 1, and then to show that the sequence is increasing. What is $\lim_{n\to\infty} a_n$?

3. Given the points $A(1,-2)$ and $B(3,3)$, find the point C on the parabola $y=x^2$, such that the sum of the squares of the distances to the points A and B is minimal.

4. Let
$$f(x)=(x+1)(x-2)$$
   and
$$g(x)=(x+1)(x-2)(x+3)^2.$$
   (a) Find all intersections of the graphs of functions f and g.
   (b) Sketch the graphs of the functions f and g.
   (c) The graphs of f and g determine three bounded sections in the plane. Determine the combined area of the three sections.

Justify your answers!