· Name and surname: ______________________ ID: __________________

Preexam for OMA, 08.01.2020

- Time limit: **30 minutes**
- For a passing grade you have to achieve at least 50% of all points. The number in the bracket [·] tells you how many points you get for a correct solution to the question.
- Any attempt of copying the solution of someone else, talking, using electronic equipment is **strictly** forbidden.

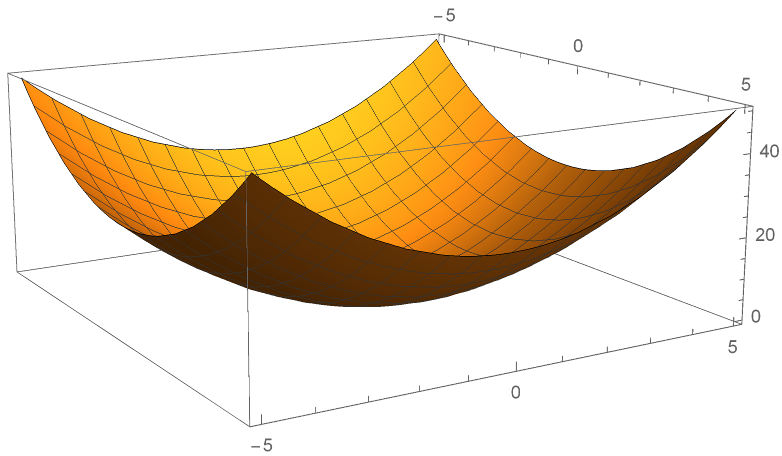
1. [**30 points**]

(a) [**10**] Write down one operation in $\mathbb{C}$ and one transformation of the complex plane, for which the polar form of a complex number is more appropriate than the cartesian form. For both of them (the operation and the transformation) write down the rule in the polar form.

(b) [**10**] Determine real coefficients $a_i \in \mathbb{R}$ in the algebraic equation $a_3z^3 + a_2z^2 + a_1z + a_0 = 0$, such that at least one solution of this equation will not be real. Justify your answer.

(c) [**10**] Determine real coefficients $a_i \in \mathbb{R}$ in the algebraic equation $a_6z^6 + a_5z^5 + \ldots + a_1z + a_0 = 0$ of degree 6, such that the set of its solutions remains the same if we rotate each solution for the angle $\frac{\pi}{6}$ in a positive direction. Justify your answer.

2. [**30 points**] On the next image there is a graph of some continuously differentiable function $f : (-5, 5) \times (-5, 5) \to \mathbb{R}$.



We notice that in the domain the function f has **exactly** one local extremum (x_0, y_0).

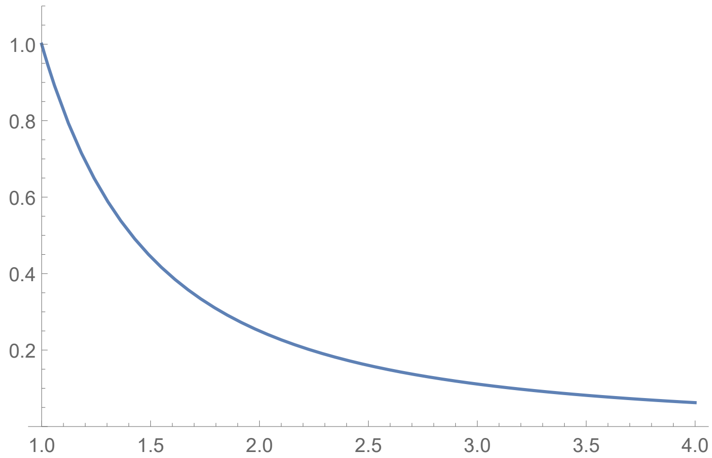
(a) [**5**] What holds for the partial derivatives $f_x(x_0, y_0)$, $f_y(x_0, y_0)$?

(b) [**10**] Write down an example of the matrix, which could be a Hessian matrix of the function f in the point (x_0, y_0). Justify your answer.

(c) [**10**] Let $g : (-5, 5) \times (-5, 5) \to \mathbb{R}$ be a continuously differentiable function. Explain how you would search the candidates for the extremal values of the function f above the curve, determined by the equation $g(x, y) = 6$.

(d) [**5**] Does there exist a function g in the previous point, such that there are infinitely many candidates in the previous point? Justify your answer.

3. [**35 points**] On the next image there is a graph of the function $f(x) = \frac{1}{x^2}$.



(a) [**8**] Calculate the definite integral $I_n := \int_1^n f(x)\,dx$ and $\lim_{n\to\infty} I_{2n}$.

(b) [**10**] Sketch the curve (not necessarily very thoroughly) and draw the rectangles that correspond to the Riemann sum of the function $f(x)$ on the interval $[1, 4]$ for the choice of the splitting points $x_0 = 1$, $x_1 = 2$, $x_2 = 3$, $x_3 = 4$ and the intermediate points $c_1 = 2$, $c_2 = 3$, $c_3 = 4$.

(c) [**10**] Write down the Riemann sum R_n of the function $f(x)$ on the interval $[1, n]$ for the choice of the splitting points $x_0 = 1, x_1 = 2, \dots, x_{n-1} = n$ of the interval $[1, n]$ and the intermediate points $c_1 = 2, c_2 = 3, \dots, c_{n-1} = n$. If you infer based on the point (3b), what can you say about the value of R_n in comparison to I_n?

(d) [**7**] Justify the fact that the series $\sum_{k=1}^{\infty} \frac{2}{k^2}$ is convergent.