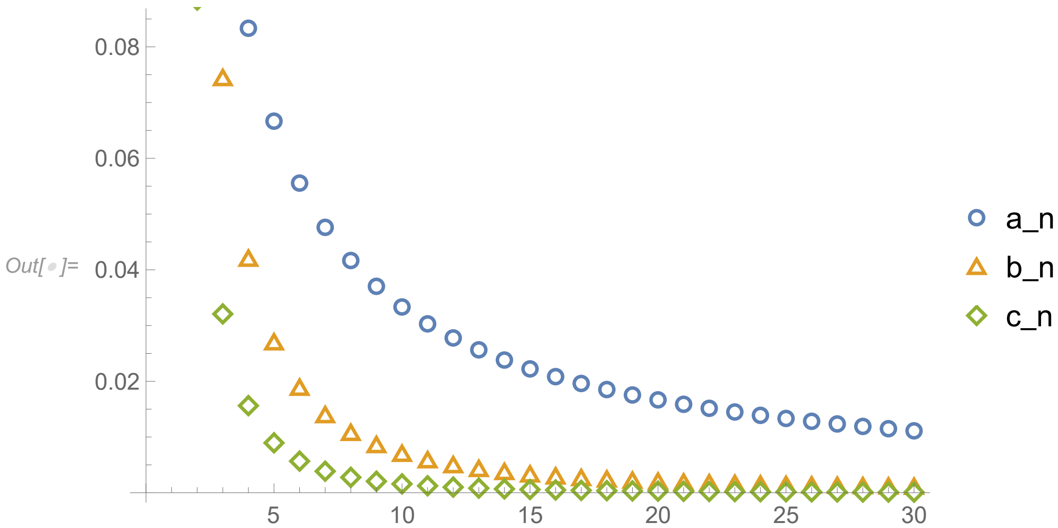
Name and surname: ______________________ ID: __________________

Exam for OMA, 02.07.2020

- Time limit: **45 minutes**
- For a passing grade you have to achieve at least 50% of all points. The number in the bracket [·] tells you how many points you get for a correct solution to the question.
- Any attempt of copying the solution of someone else, talking, using electronic equipment is **strictly** forbidden.

1. [**30 points**] There are the first 30 terms of three sequences plotted on the following image:



The sequences follow the same shape also in the continuation, i.e., the sequence a_n is above b_n, which is above c_n, all are decreasing and nonnegative.

(a) Let us assume that $\lim_{n\to\infty} a_n = 0$. What is the value of $\lim_{n\to\infty} b_n$? Is the sequence $\sum_{n=1}^{\infty}(-1)^n\, b_n$ convergent?

(b) Let us assume that $\sum_{n=1}^{\infty} b_n$ is divergent. What can you infer about the convergence/divergence of $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$?

(c) Let us assume that $\sum_{n=1}^{\infty} a_n$ divergent, while $\sum_{n=1}^{\infty} b_n$ is convergent. We build a new sequence d_n, which consists of 30 terms of the sequence c_n, then 30 terms of the sequence b_n, and then the rest are terms of the sequence a_n. What can you infer about the convergence/divergence of $\sum_{n=1}^{\infty} d_n$?

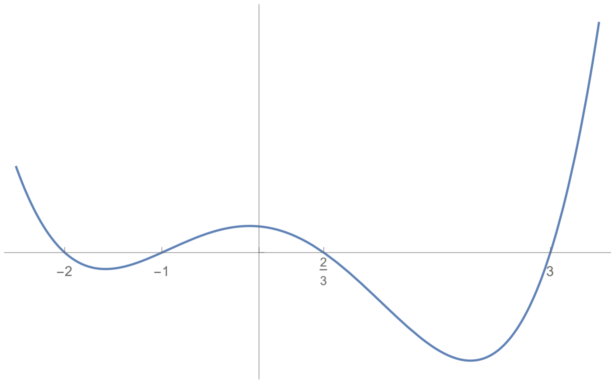
2. **[30 points]** Let $g : \mathbb{R}^2 \to \mathbb{R}$ be twice differentiable function of two variables. We have a table of function values and its partial derivatives:

(x, y)	$g(x, y)$	$g_x(x, y)$	$g_y(x, y)$	$g_{xx}(x, y)$	$g_{xy}(x, y)$	$g_{yy}(x, y)$
$(-2, 0)$	3	0	1	2	1	1
$(-1, 0)$	1	-1	0	-1	2	-1
$(\frac{2}{3}, 0)$	4	0	0	2	1	1
$(3, 0)$	5	1	0	2	1	1
$(-2, 1)$	2	0	0	-3	0	1
$(-1, 1)$	-7	2	0	2	1	1
$(\frac{2}{3}, 1)$	-1	0	0	-1	1	-2
$(3, 1)$	3	0	1	-1	-2	1

(a) Which points in the table are stationary points of g?

(b) Among points in the table above find local extrema of g and classify them.

(c) On the following image there is a graph of a function $f : [-2.5, 3.5] \to \mathbb{R}$.



Determine the constrained extrema of g on the strip $[-2.5, 3.5] \times \mathbb{R}$ at the constraint $h(x, y) = 0$, where $h(x, y) = (f(x))^2 + (y - 1)^2$.

Hint: Determine which points satisfy $h(x, y) = 0$.

3. **[40 points]**

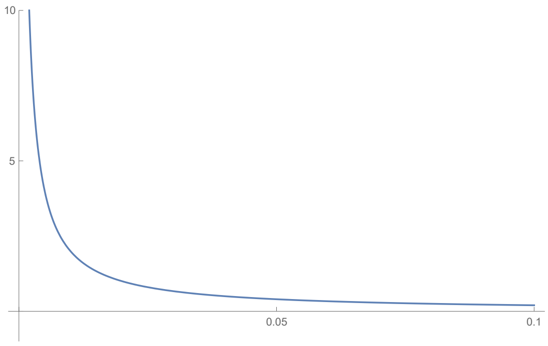


Image 1

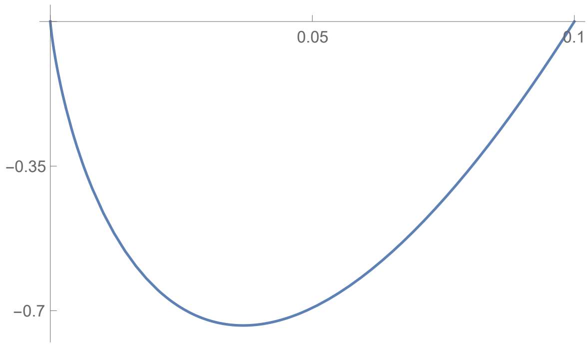


Image 2

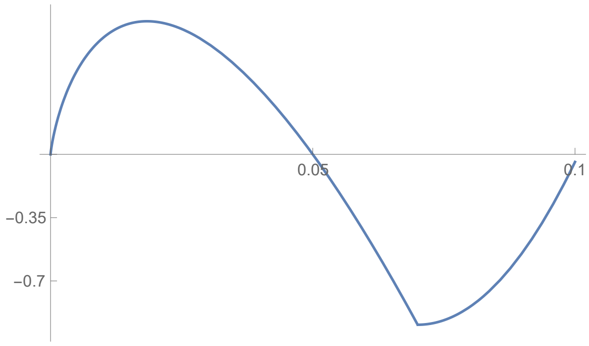


Image 3

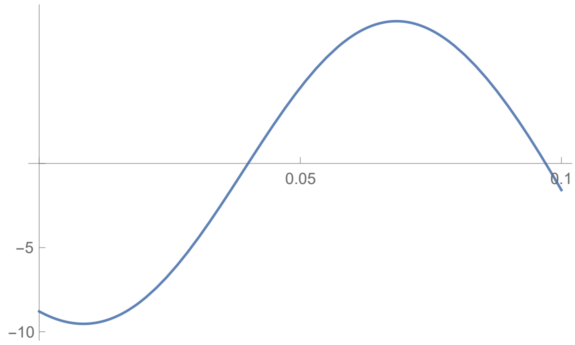


Image 4

On the images above there are graphs of some functions. For each of the following statements determine all images, which satisfy the statement and justify your decision.

(a) Definite integral $\displaystyle\int_0^{0.1} f(x)\,dx$ of the function with a graph on the image exists.

(b) Second derivative of the function with a graph on the image has at least one zero on the interval $(0, 0.1)$.

(c) First derivative of the function with a graph on the image does not exist on the whole interval $(0, 0.1)$.

(d) First derivative of the function with a graph on the image is never positive on the interval $(0, 0.1)$.