

1. kolokvij iz Matematike

(Ljubljana, 8. 12. 2016)

Time limit: 90 minutes. All problems are worth the same amount of points. Read the full text of each problem carefully. You may use two A4 sheets with formulas. The results will be available at ucilnica.fri.uni-lj.si.

Justify all your answers!

1. [25. points] Two complex numbers are given $a = i$ and $b = 1 + i$.

(a) Draw the numbers a , b^5 and ab in the complex plane.

(b) Find the number z that solves the equation $|z - a| = |\bar{b}|$ and has the smallest absolute value.

2. [20 points] The sequence is given by

$$a_n = \frac{n}{\sqrt{2 + 3n^2}}.$$

(a) Show that the sequence is increasing.

(b) Find the limit $\lim_{n \rightarrow \infty} a_n$.

3. [30 points] Function f is given by $f(x) = \sqrt{2 - x^2}$.

(a) Find the domain of definition D_f of the function f .

(b) Find zeros, stationary points, intervals of increasing and decreasing and draw the graph of f .

(c) Find the equation of the tangent to the graph f at $x = 1$.

(d) Calculate the integral

$$\int x f(x) dx.$$

4. [25 points] For three numbers a, b, c from the interval $[-2; 2]$ the following holds: the sum of first and second is 1, the sum of second and third is 2. What is the smallest value of the product abc ?

Justify all your answers!