A description of the concentration of $\mathrm{CO_2}$ in Earth's atmosphere

Hawaii's Mauna Loa observatory is used to monitor the concentration of $\mathrm{CO_2}$ in the atmosphere. The time dependence of the concetration of $\mathrm{CO_2}$ is mathematically given as a function of concentration y depending on time, $y = \mathrm{CO_2}(t)$. The mathematical model is a function of time t, also depending on parameters $\mathbf{p} = [p_1, p_2, \ldots, p_k]^\mathsf{T}$:

$$\mathrm{CO_2}(t) = F(\mathbf{p}, t) = F(p_1, p_2, \ldots, p_k, t). \tag{1}$$

A desired model is the one at which the parameters p_i have a nice interpretation in relation to the problem we'd like to understand. A model which nicely describes the concetration of $\mathrm{CO_2}$ can be made using a quadratic function – to describe the increase of $\mathrm{CO_2}$, and a periodic part – to describe yearly oscillations:

$$\mathrm{CO_2}(t) = p_1 + p_2 t + p_3 t^2 + p_4 \sin(2\pi t) + p_5 \cos(2\pi t). \tag{2}$$

Task

Write an Octave function which determines the values of parameters p_i of the linear model

$$F(\mathbf{p}, t) = p_1 f_1(t) + p_2 f_2(t) + \ldots + p_k f_k(t) \tag{3}$$

using the linear least squares method. Determine the parameters of the model (2) using the data from *Mauna Loa observatory*[1]. Graphically compare the data and the results of the model and answer these questions:

1. What average yearly concetration is predicted by the model for year 2050?
2. What is the amplitude of yearly swing of the concetration?
3. What is the yearly increase of the concetration predicted by the model for year 2050?

[1] The data is available on the website. Use daily averages. Watch for values −999 which mean that there is no data.

Detailed instructions for the Octave function

Write an Octave function `parameters` which, given a linear model $F(\mathbf{p}, t)$, determines optimal values of parameters using the linear least squares method. A function call must be of the form

```
p = parameters(x, y, model, k)
```

where:

- `p` is the column of the parameters of the model,
- `x` and `y` are columns of values of independent and dependent variables,
- `model` is a function handle describing the model. A call `y = model(p, t)` returns a value `y` which is predicted by the model using parametes `p` at point `t`,
- `k` is the number of parameters in the model.

Example

To approximate

x	1	2	3	4
y	1	2	1	2

using a line $y = a + bx$, one would use

```
model = @(p,x) p(1)+p(2)*x;
p = parametri([1;2;3;4], [1;2;1;2], model, 2)
```

Tests

The file with function `parameters` must contain tests, which will let you confirm that the program is running correctly. Below is an example of a simple test (watch the spaces after `%!`):

```
%! test
%! model = @(p, x) p(1) + p(2)*x;
%! assert(parametri([0; 1], [1; 1], model, 2), [1; 0], eps)
```

Submission

Use the online classroom to submit the following:

1. file **parameters.m** which should be well-commented and contain at least one test,

2. a report file **solution.pdf** which contains the necessary derivations and answers to questions.

As an inspiration take a look at an example of a solved task.

While you can discuss solutions of the problems with your colleagues, the programs and report must be your own creation. You can use all Octave functions from problem sessions.