Computational topology

Lab work, 8^{th} week

1. The simplicial complexes X and Y are given as lists of simplices:

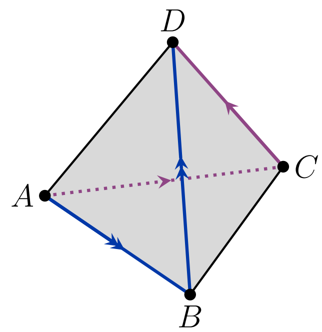
$$\begin{aligned} X &= \{A, B, C, AB, AC, BC\}, \\ Y &= \{A, B, C, D, AB, AD, BC, CD\}. \end{aligned}$$

   (a) Construct the cones CX and CY by listing all the simplices.

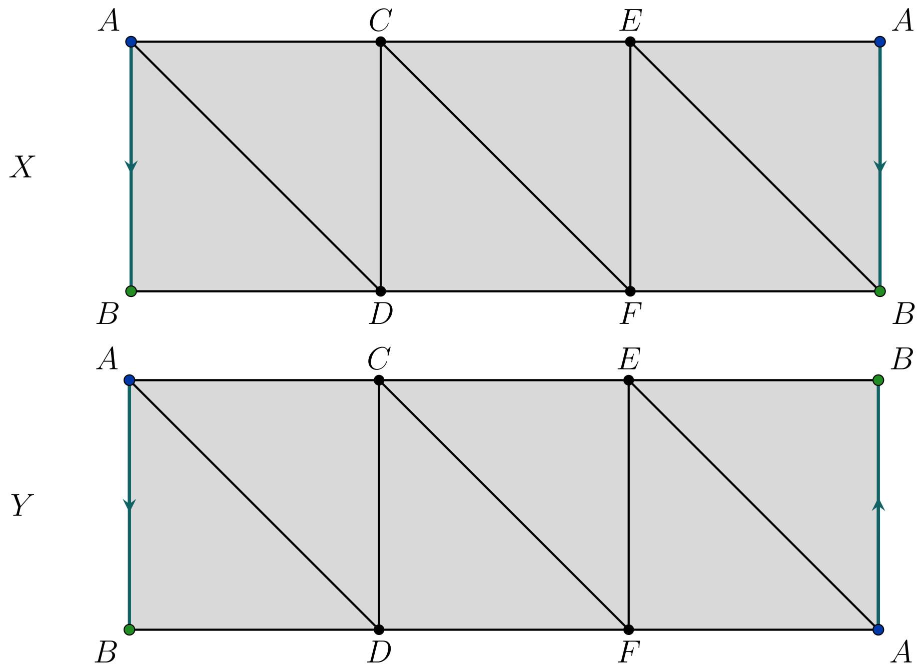
   (b) Find the sequences of collapses that simplify CX and CY as much as possible.

   (c) Is CX a collapsible complex for all X?

2. Let $X = \Delta^3$ be the standard 3-simplex (tetrahedron) with vertices A, B, C and D. We obtain Y by identifying the edges AB and BD and the edges AC and CD (preserving the ordering of vertices). Show that Y collapses onto a Klein bottle.



3. Given the following triangulations of the cylinder X and the Moebius band Y, find a sequence of elementary collapses that simplifies them as much as possible, then compute the homology groups $H_*(X)$ and $H_*(Y)$.



4. For the simplicial complex X in the figure below

(a) write down the chain groups $\mathcal{C}_n$,

(b) determine the boundary homomorphisms $\partial_n \colon \mathcal{C}_n \to \mathcal{C}_{n-1}$,

(c) find the cycles $Z_n = \ker\partial_n$,

(d) find the boundaries $B_n = \mathrm{im}\partial_n$,

(e) determine the simplicial homology groups with $\mathbb{Z}$ coefficients, $H_n(X;\mathbb{Z})$,

(f) determine the simplicial homology groups with $\mathbb{Z}_2$ coefficients, $H_n(X;\mathbb{Z}_2)$,

(g) determine the Betti numbers of X and

(h) compute the Euler characteristic of X.