



Digitalna vezja UL, FRI



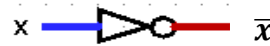
Vaja 2 Booleova algebra, logisim

1 Booleova algebra

❑ Operacije:

Negacija NE (NOT):

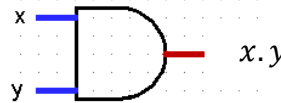
$$\bar{x} = x' = \sim x$$



| x | \bar{x} |
|---|-----------|
| 0 | 1 |
| 1 | 0 |

Konjunkcija: IN (AND)

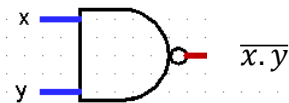
$$x \& y = x \wedge y = x.y$$



| x | y | $x.y$ | $\overline{x.y}$ |
|---|---|-------|------------------|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Negirana konjunkcija: NAND

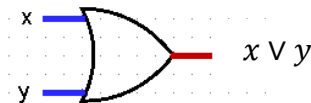
$$\overline{x.y} = x \uparrow y$$



| x | y | $x \vee y$ | $\overline{x \vee y}$ |
|---|---|------------|-----------------------|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

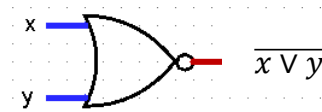
Disjunkcija: OR (ALI)

$$x \vee y = x + y$$



Negirana disjunkcija: NOR

$$\overline{x \vee y} = x \downarrow y$$



□ Zakoni in izreki

Komutativnost

$$x.y = y.x$$

$$x \vee y = y \vee x$$

Distributivnost

$$x.(y \vee z) = (x.y) \vee (x.z) = x.y \vee x.z$$

$$x \vee (y.z) = (x \vee y).(x \vee z)$$

Konstanta (0,1)

$$x \vee 0 = x$$

$$x.1 = x$$

Komplement (x, \bar{x})

$$x.\bar{x} = 0$$

$$x \vee \bar{x} = 1$$

Asociativnost - izrek

$$(x.y).z = x.(y.z) = x.y.z$$

$$(x \vee y) \vee z = x \vee (y \vee z) = x \vee y \vee z$$

Konstanta (0,1) - izrek

$$x.0 = 0$$

$$x \vee 1 = 1$$

Idempotenca - izrek

$$x.x = x$$

$$x \vee x = x$$

Vsebovanost - izrek

$$x.(x \vee y) = x$$

$$x \vee (x.y) = x$$

Dvojna negacija

$$\bar{\bar{x}} = x$$

DeMorganov izrek

$$\overline{x.y} = \bar{x} \vee \bar{y}, \quad \overline{x.y.z} = \bar{x} \vee \bar{y} \vee \bar{z}, \quad \dots$$

$$\overline{x \vee y} = \bar{x}.\bar{y}, \quad \overline{x \vee y \vee z} = \bar{x}.\bar{y}.\bar{z}, \quad \dots$$

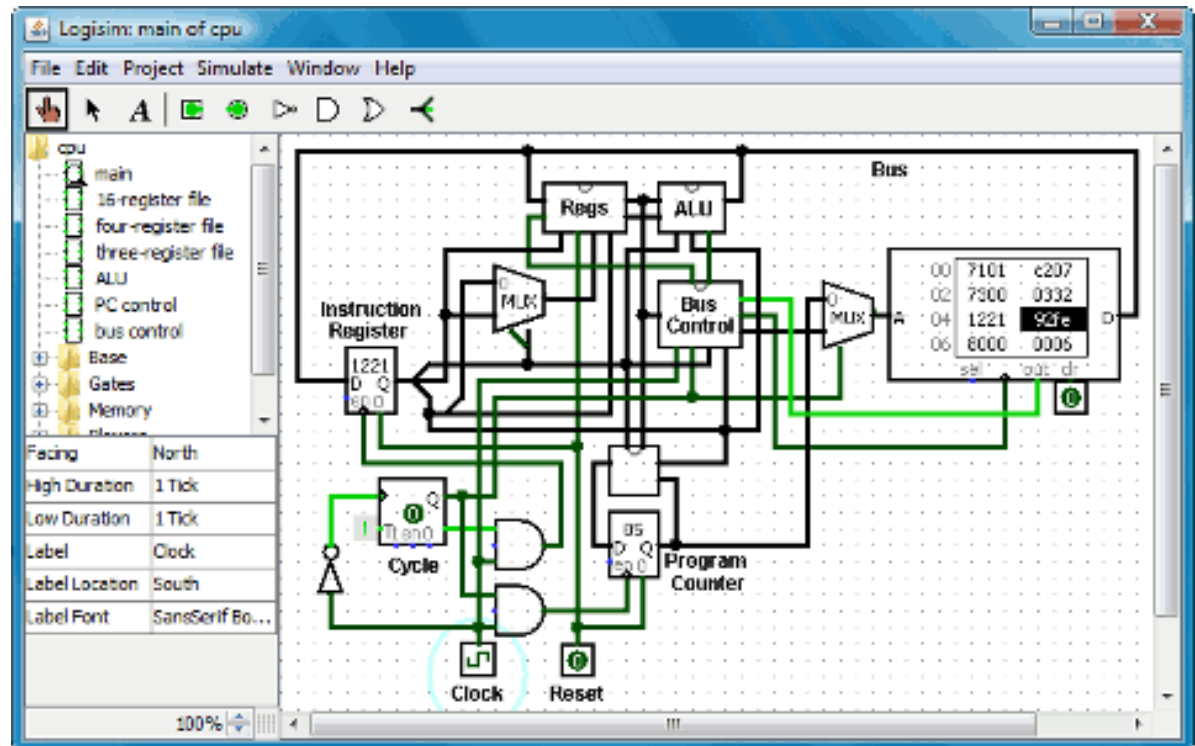
2 Logisim

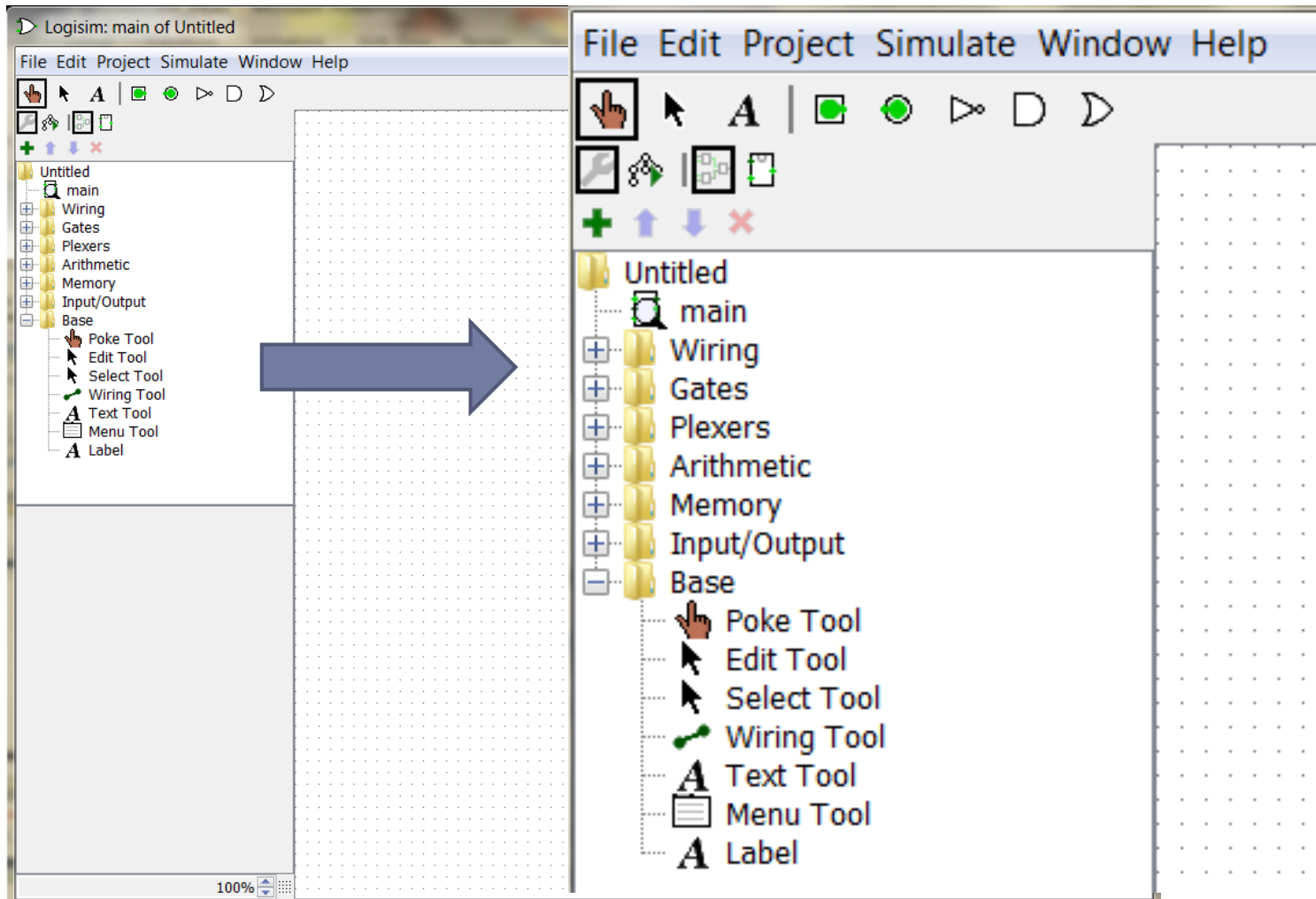
- ❑ Načrtovanje in simulacija digitalnih/logičnih vezij

Logisim, <http://www.cburch.com/logisim/index.html>

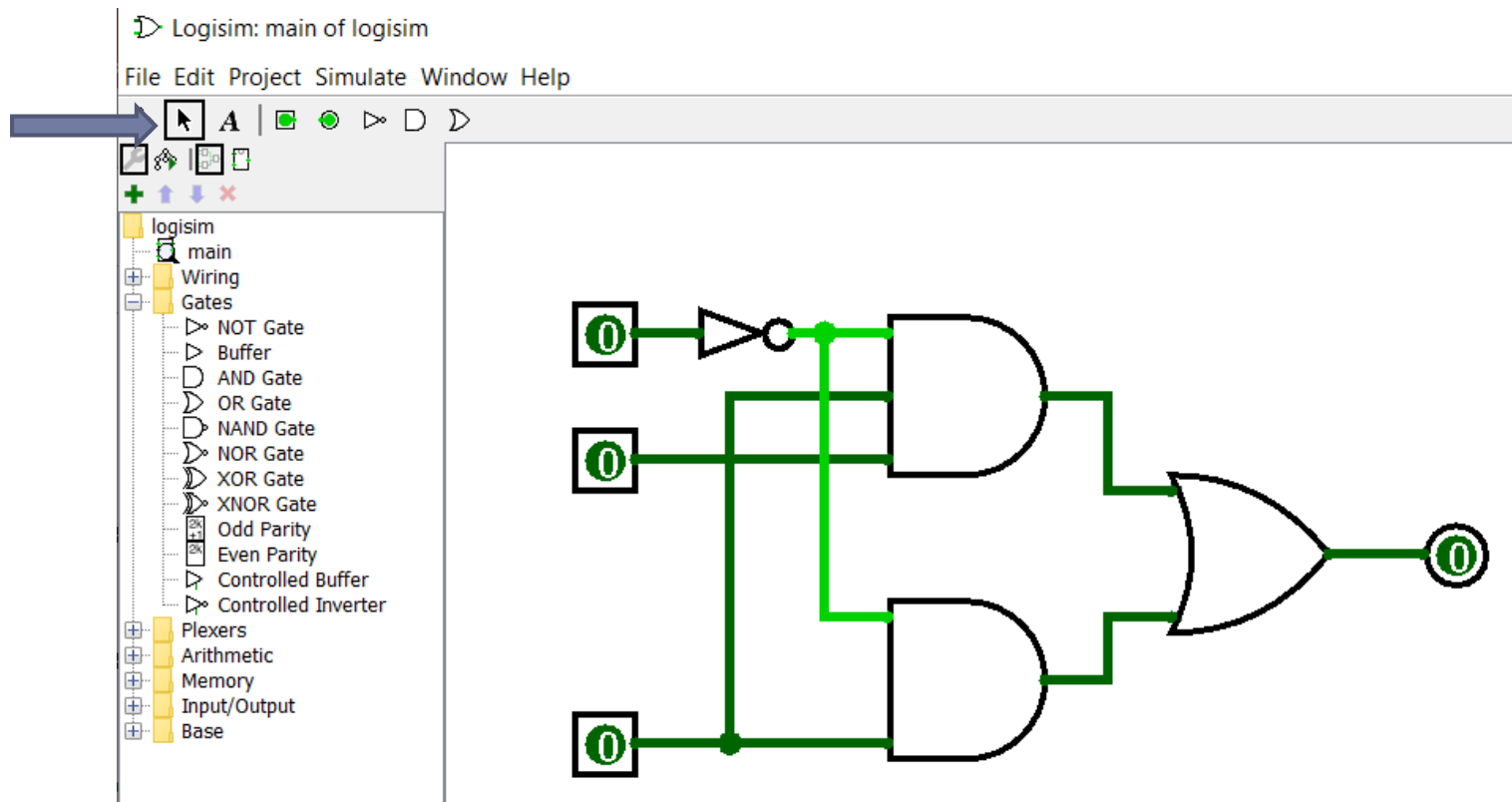
logisim-evolution, <https://sourceforge.net/projects/logisimevolution/>

logisim-win-2.7.1.exe



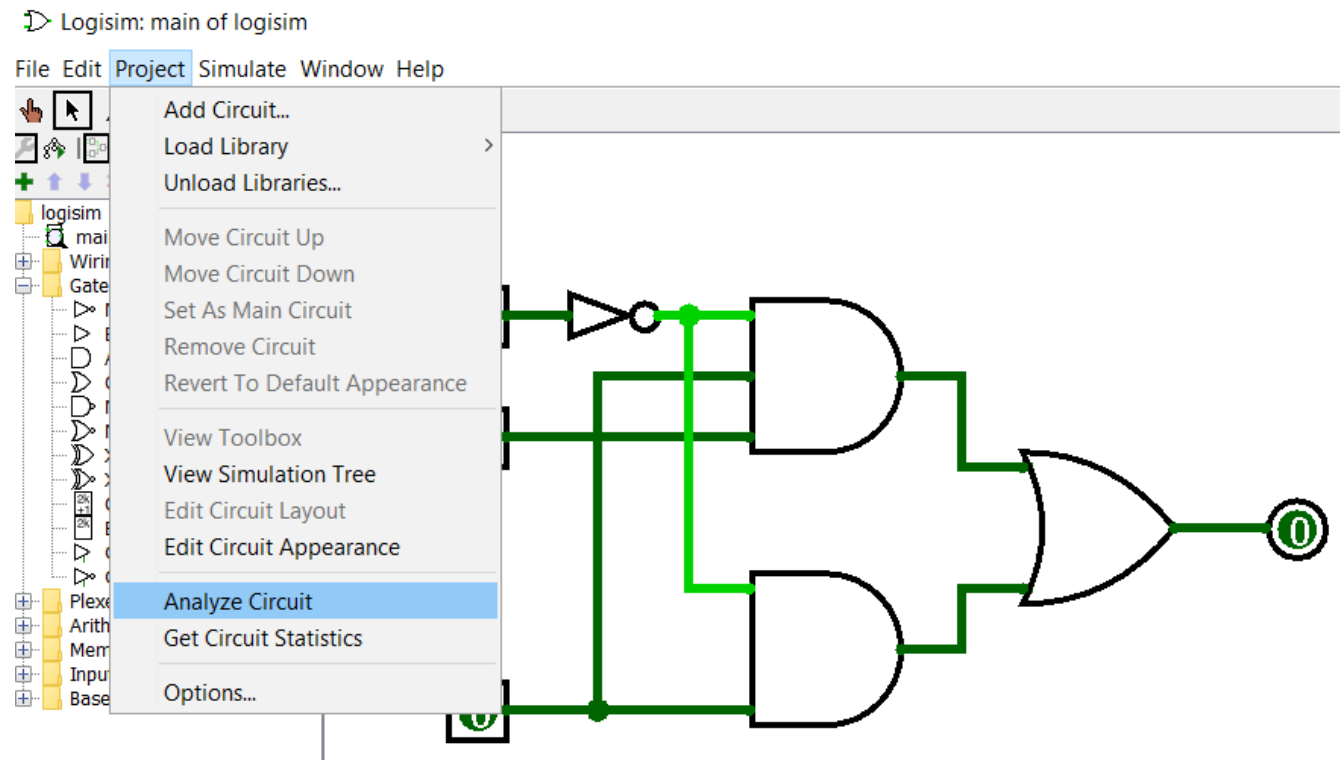


- ❑ Narišite vezje.
- ❑ Preverite izhod vezja za vse možne vhodne kombinacije.

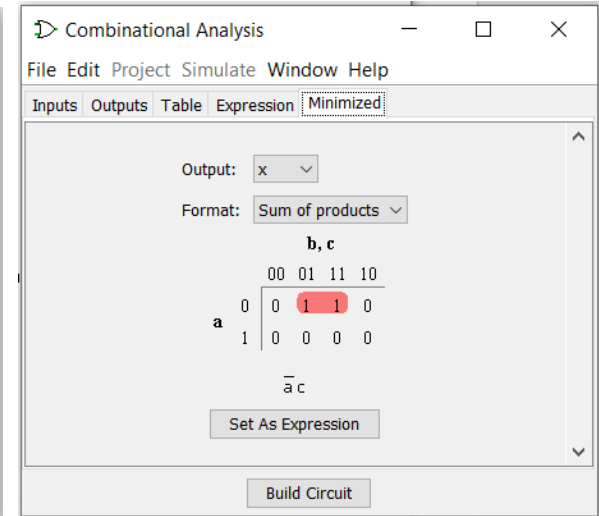
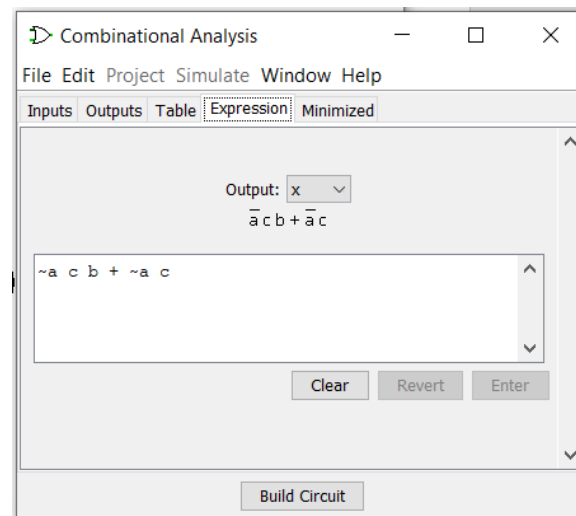
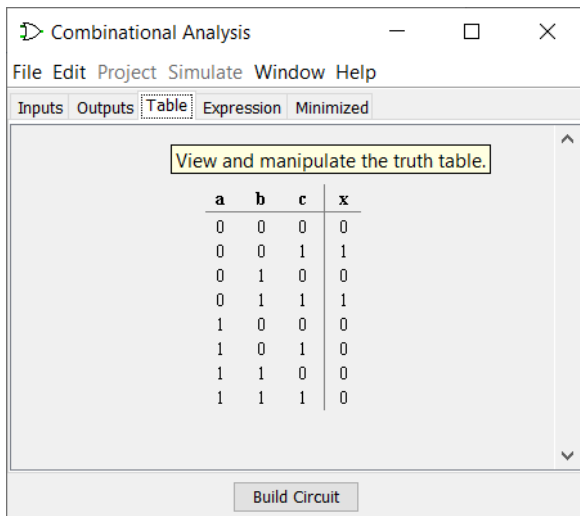
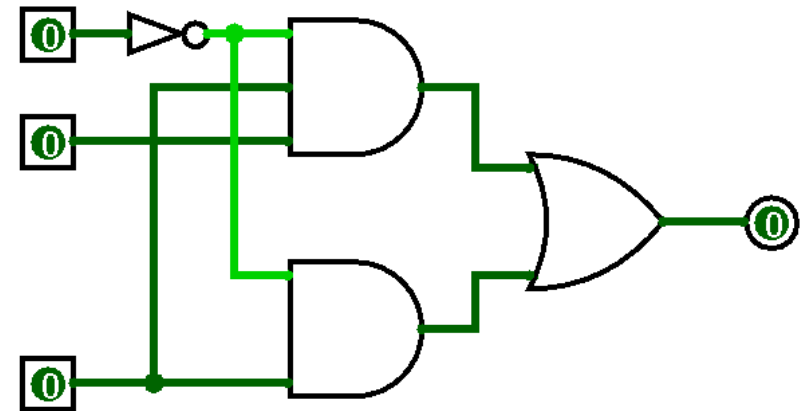


❑ Analiza vezja (ang. Analyze Circuit):

- Vhodi in izhodi
- Tabela
- Zapis funkcije
- Minimizacija



- Vhodi (a,b, c) in izhod (x) – simulator določi oznake vhodov in izhodov
- Tabela
- Zapis funkcije
- Minimizacija



Naloga 1: Poenostavljanje logičnih funkcij

❑ Poenostavljanje logičnih funkcij z uporabo Booleove algebre

❑ Primer:

$$f(x, y, z) = \bar{x} \cdot \bar{y} \cdot z \vee \bar{x} \cdot y \cdot z = \bar{x} \cdot z \cdot (\bar{y} \vee y) = \bar{x} \cdot z \cdot 1 = \bar{x} \cdot z$$

$$\begin{aligned} f(x, y, z) &= \bar{x} \cdot \bar{y} \cdot z \vee x \cdot \bar{y} \cdot \bar{z} \vee x \cdot \bar{y} \cdot z \vee \bar{x} \cdot y \cdot z = \\ &= \bar{x} \cdot z(\bar{y} \vee y) \vee x \cdot \bar{y} \cdot (\bar{z} \vee z) \\ &= \bar{x} \cdot z \vee x \cdot \bar{y} \end{aligned}$$

❑ Poenostavite podane logične funkcije.

1. $f(x, y, z) = \bar{x} \cdot \bar{y} \cdot z \vee x \cdot y \cdot z \vee x \cdot y \cdot \bar{z} \vee x \cdot \bar{y} \cdot z = ?$

2. $f(x, y, z, u) = \bar{x} \cdot \bar{y} \cdot z \cdot u \vee x \cdot y \cdot z \cdot u \vee x \cdot y \cdot \bar{z} \cdot u \vee x \cdot y \cdot z \cdot \bar{u} = ?$

3. $f(x, y) = \overline{\bar{x} \cdot \bar{y}} \vee \overline{x \cdot y} = ?$

4. $f(x, y, z) = \overline{(\bar{x} \cdot \bar{y} \vee y \cdot z)} \vee (x \vee z) = ?$

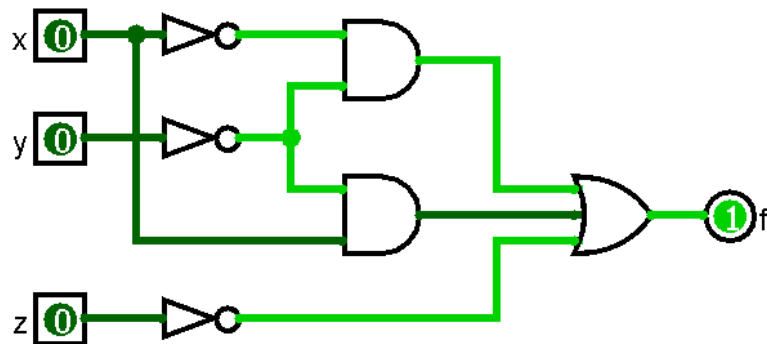
5. $f(A, B, C, D) = A \cdot C \vee \overline{(\bar{A} \cdot \bar{B} \vee A \cdot B)} \cdot (\bar{C} \vee \overline{A \cdot D}) = ?$

6. $f(A, B, C, D) = A \vee C \cdot \overline{(\bar{A} \cdot \bar{B} \vee B)} \vee \bar{C} \cdot \overline{B \vee \bar{D}} = ?$

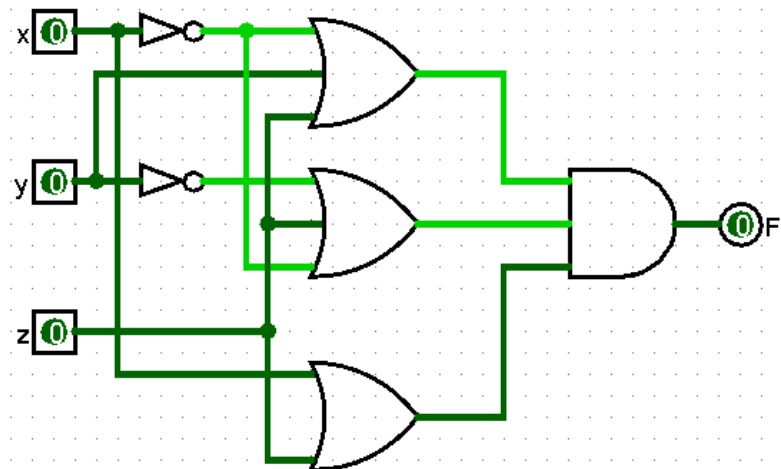
Naloga 2

Zapišite izhod podanih logičnih vezij z operatorji NOT, AND, OR

1) $f(x,y,z) = ?$



2) $F(x,y,z) = ?$



Naloga 3

1. V podano pravilnostno tabelo zapišite logično funkcijo - varianta a) ali b):

$$f(A, B, C) = A \cdot \bar{B} \vee \overline{A \cdot B} \vee A \cdot B \cdot C$$

2. Narišite vezje v logisimu in preverite rezultat.

a)

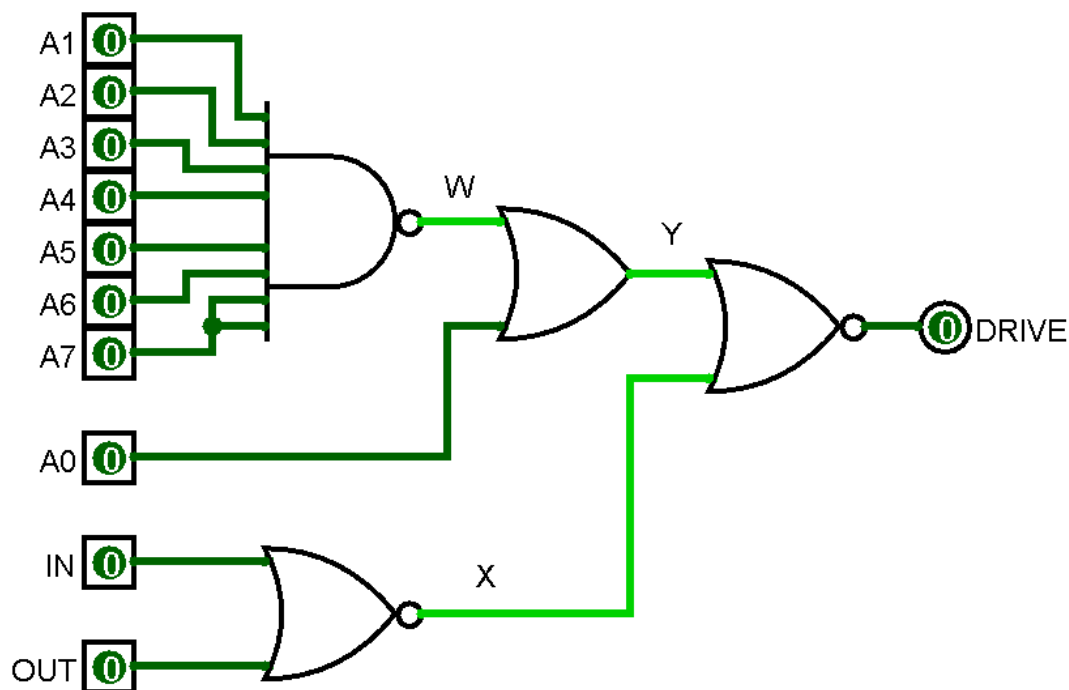
| A | B | C | $A \cdot \bar{B}$ | $\overline{A \cdot B}$ | $A \cdot B \cdot C$ | $f(A, B, C)$ |
|---|---|---|-------------------|------------------------|---------------------|--------------|
| 0 | 0 | 0 | | | | |
| 0 | 0 | 1 | | | | |
| 0 | 1 | 0 | | | | |
| 0 | 1 | 1 | | | | |
| 1 | 0 | 0 | | | | |
| 1 | 0 | 1 | | | | |
| 1 | 1 | 0 | | | | |
| 1 | 1 | 1 | | | | |

b)

| A | B | C | $f(A, B, C)$ |
|---|---|---|--------------|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

Naloga 4

Podano logično vezje se uporablja za nadzor motorja pogonskega vretena za pogon diskete, ko mikroračunalnik pošilja podatke ali jih prejema z diskete. Vezje bo vklopilo motor, ko bo signal DRIVE enak 1. Določite vhodne pogoje, ki so potrebni za vklop motorja tako, da interpretirate vrednosti signalov Y, X in W.



Domača naloga 1

Logične funkcije zapišite z novim naborom operatorjev. Uporabite zakone in izreke Booleove algebre. Rezultat preverite v logisimu tako, da realizirate logično vezje in preverite enakost v tabeli.

Rešitve zapišite na list A4 in prinesite na naslednje vaje.

1) Pretvorba v zapis z operatorji AND, OR in negiranimi spremenljivkami:

$$f(A, B, C) = \overline{A} \cdot \overline{B} \vee \overline{A} \cdot \overline{B} \vee \overline{A} \cdot B \cdot C$$

$$f(A, B, C, D) = \overline{\overline{A \vee B} \vee (\overline{B \vee C})} \cdot D$$

2) Pretvorba v zapis z operatorji NAND

$$f(A, B, C) = A \vee B \cdot \overline{C}; \quad f(A, B, C) = A \cdot C \vee \overline{B} \cdot \overline{C}$$

3) Pretvorba v zapis z operatorji NOR

$$f(A, B, C) = (A \vee C) \cdot \overline{B}; \quad f(A, B, C) = (A \vee \overline{B}) \cdot (B \vee \overline{C})$$

4) Poenostavitev logične funkcije z uporabo Booleove algebre

$$f(A, B, C, D) = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} \vee \overline{A} \cdot B \cdot \overline{C} \cdot \overline{D} \vee \overline{A} \cdot B \cdot C \cdot \overline{D} \vee A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} \vee \\ A \cdot B \cdot \overline{C} \cdot \overline{D} \vee A \cdot \overline{B} \cdot \overline{C} \cdot D \vee A \cdot B \cdot C \cdot \overline{D} \vee \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D$$