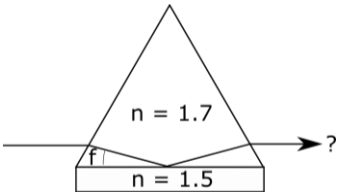
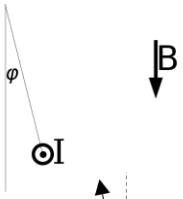
1. izpit iz fizike - 17. 1. 2022
10:15-11:45, oddaja do 12:05. Podaljšan čas pisanja do 12:30, oddaja 12:50
English version below.

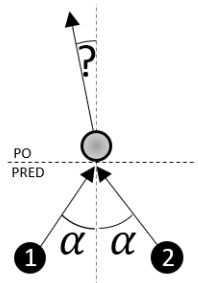
1.) Prizmo v obliki enakostraničnega trikotnika z lomnim količnikom $n = 1.7$ položimo na steklo z lomnim količnikom $n = 1.5$. Na prizmo svetimo s svetlobo v smeri vzporedno glede na stično ravnino (glej skico). Pod kolikšnim kotom (f, glej skico) zadane spodnjo stranico prizme? Ali se svetloba popolno odbije na stični površini, ali se lomi v steklo?



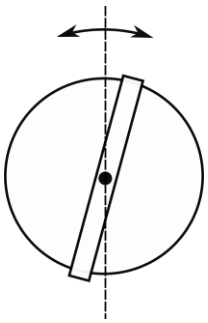
2.) Vodoraven aluminijast trak obesimo na dve lahki vrvici in ga kot gugalnico postavimo v navpično usmerjeno magnetno polje $B = 1\,T$. Ko po traku poženemo tok, se trak odkloni za kot $\varphi = 5°$. Kolikšen je električen tok po traku? Presek traku je $S = 2\,mm^2$, dolžina $l = 1\,m$, gostota aluminija pa $2700\ kg/m^3$.



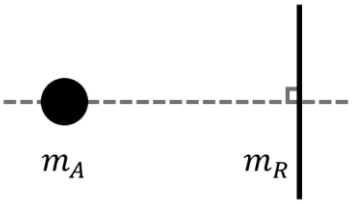
3.) Dve plošči drsita po ledu brez trenja in trčita ter se zlepita (skica). Vpadni kot obeh plošč je $\alpha = 30°$. Hitrost prve plošče je $4\,m/s$ in druge plošče je $2\,m/s$. Masa druge plošče je trikrat večja od prve. Izračunaj velikost hitrosti ter kot kamor odleti zlepek po trku. Kolikšen delež začetne mehanske energije se pri trku izgubi?



4.) V kompasu ima magnetna igla v obliki palice dolžino $l = 10\,cm$, maso $m_1 = 30\,g$ in dipolni moment $p = 8\,Am^2$. Igla je pritrjena na disk, ki ima maso $m_2 = 60\,g$, in radij $r = 5\,cm$. V težišču je igla vpeta na os okoli katere se lahko prosto vrti. Kompas položimo na ravno mizo in počakamo da se igla ustali. Nato v ravnini mize in pod kotom $10°$ glede na smer magnetnega severnega tečaja vključimo homogeno magnetno polje $B = 5\,mT$. S kolikšno frekvenco in s kolikšno amplitudo igla zaniha, če vpliv Zemeljskega magnetnega polja na nihanje zanemarimo? Določi kotno hitrost igle ob času $t = 0.1\,s$ po vklopu polja.

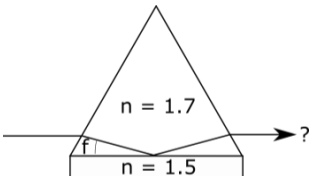


5.) Asteroid se v vesolju znajde $1\,km$ stran od rakete. Zveznica, ki povezuje asteroid in sredino rakete je pravokotna na raketo. Raketo lahko obravnavamo kot palico, ki je dolga $150\,m$ in težka $500\,t$, asteroid pa kot točko z maso $10\,t$. S kolikšno silo deluje raketa na asteroid?

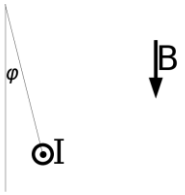


1. exam in physics - 17. 1. 2022

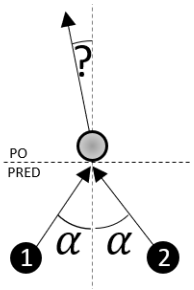
1.) A prism in the shape of an equilateral triangle with refractive index $n = 1.7$ is placed on a glass with refractive index $n = 1.5$. We shine the light on the prism in a direction parallel to the plane of contact (see sketch). At what angle (f, see sketch) does it strike the lower side of the prism? Is the light completely reflected at the contact surface, or does the light refract into the glass?



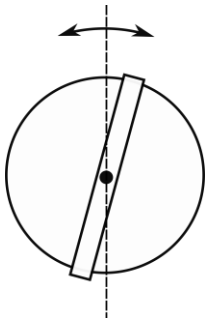
2.) Horizontal Aluminum wire is hanging on two light threads as a swing (see sketch). This is placed in a vertical magnetic field $B = 1\,T$. The current flows in the wire, the swing tilts for $\varphi = 5°$. Calculate the current in the wire. The wire cross section is $S = 2\,mm^2$, wire length is $l = 1\,m$ and Aluminum density is $2700\,kg/m^3$.



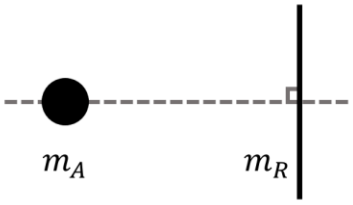
3.) Two plates slide on ice without friction. They collide and stick together (sketch). The angle of incidence of both plates is $\alpha = 30°$. The speed of the first plate is $4\,m/s$ and the speed of the second plate is $2\,m/s$. The mass of the second plate is three times greater than the first. Calculate velocity (both magnitude and direction) of the merged plates after the collision. What fraction of the initial mechanical energy is lost during the collision?



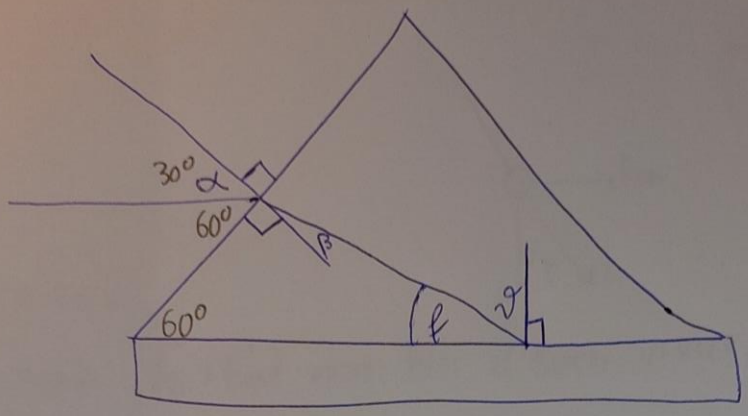
4.) In a compass, a magnetic needle in the form of a rod has a length of $l = 10\,cm$, a mass of $m_1 = 30\,g$ and a dipole moment of $p = 8\,Am^2$. The needle is attached to a disc with mass $m_2 = 60\,g$, and radius $r = 5\,cm$. At the centre of gravity the needle is fixed on an axis about which it is free to rotate. We place the compass on a flat table and allow the needle to settle. Then, in the plane of the table and at an angle of $10°$ to the direction of the magnetic north pole, we apply a homogeneous magnetic field $B = 5\,mT$. With what frequency and amplitude does the needle oscillate around the new equilibrium direction? You can neglect the influence of the Earth's magnetic field on the oscillation. Determine the angular velocity of the needle at time $t = 0.1$ s after the field is switched on.



5.) An asteroid in space is $1\,km$ away from the rocket. The line connecting the asteroid and the centre of the rocket is perpendicular to the rocket. The rocket can be thought of as a rod $150\,m$ long and $500\,t$ heavy and the asteroid as a point body with a mass of $10\,t$. How much force does the rocket exert on the asteroid?



7)



$\sin\beta = \sin\alpha \frac{n_1}{n_2}$ (5)

$n_1 = 1$ (zrak)

$n_2 = 1{,}7$ (prizma)

$\alpha = 30°$

$\Rightarrow \beta = 17{,}7°$ (5)

$60° + 90° + \beta + f = 180°$ (trikotnik na skici)

$f = 30° - \beta = 12{,}9°$ (5)

Totalni odboj

$\vartheta_{max} = \arcsin\left(\frac{n_2}{n_1}\right) = 67{,}93°$ (5)

$\Rightarrow f_{max} = 90° - 67{,}93° = 28{,}07°$

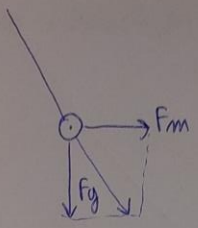
$f_{max} > f$, torej je popolni odboj. (5)

2)

$F_m = I \ell B$ (5)

smer $\rightarrow \vec{F}_m$ (5)

$m = S \cdot \ell \cdot \rho$ (5)



2 načina reševanja:

a) rezultanta $(\vec{F}_g + \vec{F}_m)$ mora biti v smeri vrvice, ker lahko vrvica deluje samo vzdolž vrvice.

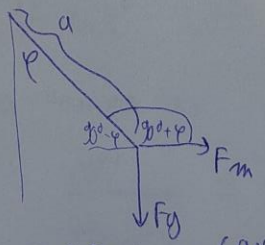
Torej $\tan 5° = \frac{F_m}{F_g}$ (5) $\frac{I \ell B}{S \cdot \ell \cdot \rho \, g} = \frac{IB}{S\rho g}$

$S \cdot \ell \cdot \rho$: Volumen, gostota, masa

$$I = \frac{S\rho g}{B} \tan(5°) = 4{,}6 \text{ mA} \quad (5)$$

b) Ravnovesje navorov. $M_g + M_m = 0$

Os izberemo v primejališču vrvice.



$a \, m g \sin(180° - \varphi) = I \ell B \, a \sin(90° + \varphi)$ (5)

$S S \ell \, g \sin(180° - \varphi) = I \ell B \sin(90° + \varphi)$

$\sin(180° - \varphi) = \sin(\varphi)$

$\sin(90° + \varphi) = \cos(\varphi)$

$$\frac{S\rho g}{B} \frac{\sin(\varphi)}{\cos(\varphi)} = I = 4{,}6 \text{ mA} \quad (5)$$

3) Ohranja se Γ. Ne ohranja se W.

$v_1 = 4\frac{m}{s}$

$v_2 = 2\frac{m}{s}$ (predznak upoštevamo v enačbah)

$m_2 = 3m_1$

x smer: $\Gamma_{pred} = \Gamma_{potem}$

$\Gamma_{pred} = m_1 v_1 \sin(30°) - 3m_1 v_2 \sin(30°)$

$\Gamma_{po} = 4m_1 v_x$

$v_x = \frac{v_1 \sin(30°) - 3 \cdot v_2 \sin(30°)}{4} = -0{,}25\frac{m}{s}$ (5)

y smer: $\Gamma_{pred} = (m_1 v_1 + 3m_1 v_2)\cos(30°) = \Gamma_{po} = 4m_1 v_y$

$v_y = \frac{v_1 + 3v_2}{4}\cos(30°) = 2{,}165\frac{m}{s}$ (5)

celotna $|v| = \sqrt{v_x^2 + v_y^2} = 2{,}18\frac{m}{s}$ (5)

smer: $\varphi = \arctan\left(\frac{v_x}{v_y}\right) = 6{,}6°$ (5)

Sprememba energije: Oziroma zanima nas

$$\frac{-W_{končna} + W_{začetna}}{W_{začetna}} = \frac{\Delta W}{W_{začetna}} = \frac{-4v^2 + v_1^2 - 3v_2^2}{v_1^2 + 3v_2^2} =$$

$W_z = \frac{m_1 v_1^2}{2} + \frac{3m_1 v_2^2}{2}$

$W_k = 4m_1 \frac{v^2}{2}$

$= -4\frac{v^2}{v_1^2 + 3v_2^2} + 1 =$

$= 1 - 0{,}68 =$ ~~68%~~ 32% (5)

4) Imamo nihanje okoli osi. Torej rešujemo enačbo $M = J\alpha = J\ddot{\varphi}$

$J = J_1 + J_2 = \underbrace{\frac{1}{12} m_1 \ell^2}_{\text{palica}} + \underbrace{\frac{1}{2} m_2 r^2}_{\text{disk}} = 10^{-4}\ \text{kg m}^2$ (5)

$\vec{M} = \vec{p}_m \times \vec{B}$ zanima nas samo $|M|$

$M = pB \cdot \sin\varphi \approx pB\varphi$

$pB\varphi = J\ddot{\varphi} \Rightarrow \omega = \sqrt{\frac{pB}{J}} = 20\ \text{Hz}$ (5)

$\nu = \frac{20}{2\pi}\ \text{Hz} = 3{,}18\ \text{Hz}$

Ob $t=0$ je

$\varphi(t=0) = 10° = A \cdot \cos(\omega t + \delta) = A \cdot \cos(\delta)$

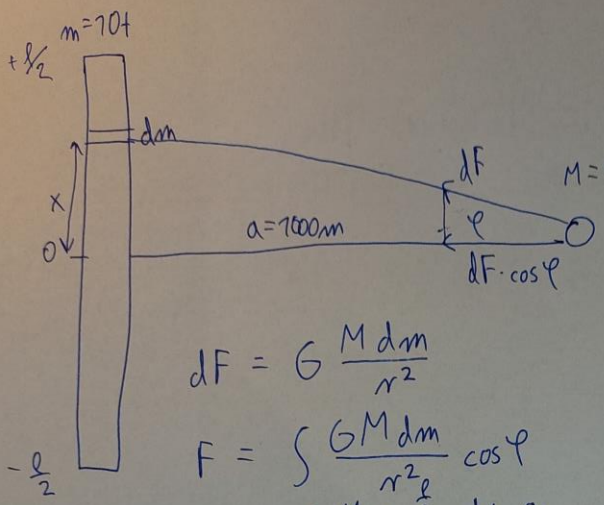
$\omega(t=0) = 0 = -A\omega \sin(\omega t + \delta) = -A\omega \sin(\delta)$

$\Rightarrow \delta = 0°$ (5)

$A = 10° = 0{,}1745\ \text{rad}$

$\omega(t = 0{,}1\,\text{s}) = -A\omega \sin(\omega t) = -3{,}17\ \text{l/s}$ (5)

5)



$\tan\varphi = \frac{x}{a}$

$\cos\varphi = \frac{a}{\sqrt{a^2+x^2}}$

$r = x^2+a^2$

$dm = \frac{m}{\ell}dx$

$$dF = G\frac{M\,dm}{r^2}$$

$$F = \int \frac{GM\,dm}{r^2}\cos\varphi$$

$$= G\frac{Mm}{L}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{dx\cdot a}{(x^2+a^2)^{3/2}}$$

(70)

substitucija: $x = a\cdot\tan(u)$ $\quad u = \operatorname{actan}\left(\frac{x}{a}\right)$

$$dx = \frac{a\,du}{\cos^2 u}$$

$$F = G\frac{Mma}{\ell}\int_{-\arctan\left(\frac{\ell}{2a}\right)}^{\arctan\left(\frac{\ell}{2a}\right)} \frac{a}{a^3}\frac{du}{\cos^2(u)}\frac{1}{(\tan^2 u+1)^{3/2}}$$

$$\sqrt{\tan^2 u+1} = \frac{1}{\cos u}$$

$$F = G\frac{Mm}{\ell a}\int_{-\arctan\left(\frac{\ell}{2a}\right)}^{\arctan\left(\frac{\ell}{2a}\right)} \cos(u)\,du =$$

$$= G\frac{Mm}{\ell a}\sin u\Big|_{-\arctan\left(\frac{\ell}{2a}\right)}^{\arctan\left(\frac{\ell}{2a}\right)}$$

iz tabele: $\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$

$$= \frac{2GMm}{\ell a\cdot 2a}\frac{\ell}{\sqrt{1+\left(\frac{\ell}{2a}\right)^2}} = G\frac{Mm}{a^2}\frac{1}{\sqrt{1+\left(\frac{\ell}{2a}\right)^2}}$$

15 toč, razdelitev odvisna od točnega postopka