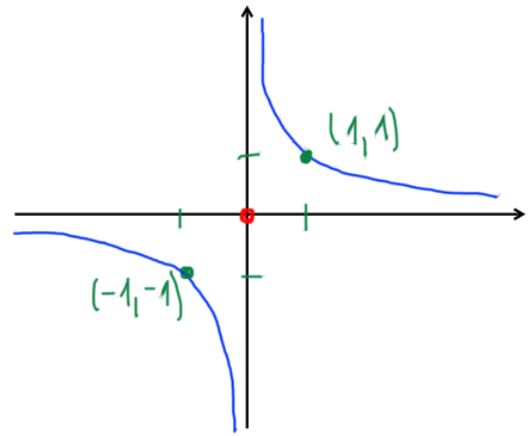
OMA vaje 11

(1.) $(x,y) = ?$ $\quad y = \frac{1}{x}$ in $f(x,y) = x^2 + y^2 \longrightarrow \min$



$\boxed{\text{1. NAČIN}}$ $\quad F(x) = x^2 + \frac{1}{x^2} \longrightarrow \min$

$$F'(x) = 2x - 2\cdot\frac{1}{x^3} = 2\cdot\frac{x^4-1}{x^3} = 0 \Rightarrow \quad x^4 - 1 = 0$$

$$x^4 = 1$$

$$F''(x) = 2 + 6\cdot\frac{1}{x^4} > 0, \ \forall x$$

$$x = \pm 1 \qquad y = \pm 1$$

MIN $(1,1)$ in $(-1,-1)$

Upazimo, da je min $f(x,y) = x^2 + y^2$ v točki $(0,0)$.

Dobili smo točki, ki zadoščata pogoju $y = \frac{1}{x}$ in sta najbližji $(0,0)$.

$\boxed{\text{2. NAČIN}}$ Poiščemo vezani ekstrem.

LAGRANGEOVA FUNKCIJA:

$$\boxed{L(x,y,\lambda) = \underbrace{f(x,y)}_{\text{funkcija}} - \underbrace{\lambda}_{\text{parameter}}\, \underbrace{g(x,y)}_{\text{pogoj}}}$$

$$L(x,y,\lambda) = x^2 + y^2 - \lambda \cdot \left(y - \frac{1}{x}\right)$$

- $L_x(x,y,\lambda) = 2x - \lambda \cdot \frac{1}{x^2} = \frac{2x^3 - \lambda}{x^2} = 0 \Rightarrow 2x^3 - \lambda = 0 \Rightarrow \lambda = 2x^3$

- $L_y(x,y,\lambda) = 2y - \lambda = 0 \Rightarrow \lambda = 2y$

$$2x^3 = 2y$$

$$\boxed{x^3 = y}$$

- $L_\lambda(x,y,\lambda) = -y + \frac{1}{x} = 0$

$$-x^3 + \frac{1}{x} = 0$$

$$\frac{-x^4 + 1}{x} = 0 \Rightarrow 1 - x^4 = 0$$

$$x^4 = 1$$

$$x = \pm 1 \quad y = \pm 1 \qquad (1,1) \text{ in } (-1,-1)$$

HESSEJEVA MATRIKA:

$$\left.\begin{array}{l} f_x = 0 \\ f_y = 0 \end{array}\right\} \text{s.t.}$$

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

- $\det(H(s.t.)) > 0$ in $f_{xx} > 0 \Rightarrow$ MIN v s.t.
- $\det(H(s.t.)) > 0$ in $f_{xx} < 0 \Rightarrow$ MAX v s.t.
- $\det(H(s.t.)) < 0$ SEDLO v s.t.

3. a) $f(x,y) = 2x^3 + 6xy^2 - 3y^3 - 150x$

$f_x(x,y) = 6x^2 + 6y^2 - 150 = 0 \Rightarrow 6(x^2 + y^2 - 25) = 0$

$f_y(x,y) = 12xy - 9y^2 = 0 \Rightarrow 3y(4x - 3y) = 0$

$y = 0$ ALI $y = \frac{4x}{3}$

$x^2 - 25 = 0$

$x = \pm 5$

$x^2 + \frac{16x^2}{9} - 25 = 0 \quad / \cdot 9$

$9x^2 + 16x^2 = 25 \cdot 9$

$\cancel{25}x^2 = \cancel{25} \cdot 9$

$x^2 = 9$

$x = \pm 3 \quad y = \pm 4$

S.T. $(5,0)$ $(3,4)$

$(-5,0)$ $(-3,-4)$

$f_{xx} = 12x$

$f_{xy} = 12y$

$f_{yx} = 12y$

$f_{yy} = 12x - 18y$

$$H = \begin{bmatrix} 12x & 12y \\ 12y & 12x-18y \end{bmatrix}$$

- $\det(H(5,0)) = \begin{vmatrix} 60 & 0 \\ 0 & 60 \end{vmatrix} = 3600 > 0$ in $f_{xx} = 60 > 0$ MIN v $(5,0)$
- $\det(H(-5,0)) = \begin{vmatrix} -60 & 0 \\ 0 & -60 \end{vmatrix} = 3600 > 0$ in $f_{xx} = -60 < 0$ MAX v $(-5,0)$
- $\det(H(3,4)) = \begin{vmatrix} 36 & 48 \\ 48 & -36 \end{vmatrix} = -3600 < 0$ SEDLO v $(3,4)$
- $\det(H(-3,-4)) = \begin{vmatrix} -36 & -48 \\ -48 & 36 \end{vmatrix} = -3600 < 0$ SEDLO v $(-3,-4)$

b) $f(x,y) = x^4 + y^4 - 36xy$

$f_x(x,y) = 4x^3 - 36y = 4(x^3 - 9y) = 0 \quad \Rightarrow \quad x^3 = 9y \;/^3 \quad \Rightarrow \quad x^9 = 9^3 \cdot y^3$

$f_y(x,y) = 4y^3 - 36x = 4(y^3 - 9x) = 0 \quad \Rightarrow \quad y^3 = 9x$

$x^9 = 9^3 \cdot 9x$

$x(x^8 - 9^4) = 0$

$x(x^8 - 3^8) = 0$

$x = 0 \qquad y = 0$

$x = 3 \qquad y = 3$

$x = -3 \qquad y = -3$

S.T. $(0,0), (3,3), (-3,-3)$

$$f_{xx} = 12x^2$$
$$f_{xy} = -36$$
$$f_{yx} = -36$$
$$f_{yy} = 12y^2$$

$$H = \begin{bmatrix} 12x^2 & -36 \\ -36 & 12y^2 \end{bmatrix}$$

- $\det(H(0,0)) = \begin{vmatrix} 0 & -36 \\ -36 & 0 \end{vmatrix} = -1296 < 0$ SEDLO v $(0,0)$
- $\det(H(3,3)) = \begin{vmatrix} 108 & -36 \\ -36 & 108 \end{vmatrix} > 0$ in $f_{xx} = 108 > 0$ MIN v $(3,3)$
- $\det(H(-3,-3)) = \begin{vmatrix} 108 & -36 \\ -36 & 108 \end{vmatrix} > 0$ in $f_{xx} = 108 > 0$ MIN v $(-3,-3)$

4. $f(x,y) = x \cdot y$, min in max na krogu $x^2 + y^2 \leq 1$

$$\boxed{L(x,y,\lambda) = x \cdot y - \lambda(x^2 + y^2 - 1)}$$

$$L_x(x,y,\lambda) = y - 2\lambda x = 0 \Rightarrow y = 2\lambda x$$

$$L_y(x,y,\lambda) = x - 2\lambda y = 0 \Rightarrow x - 2\cdot\lambda\cdot 2\lambda x = 0$$

$$x \cdot (1 - 4\lambda^2) = 0 \Rightarrow \boxed{x = 0 \quad y = 0}$$

$$1 - 4\lambda^2 = 0$$

$$\lambda = \pm \frac{1}{2} \Rightarrow \boxed{y = \pm x}$$

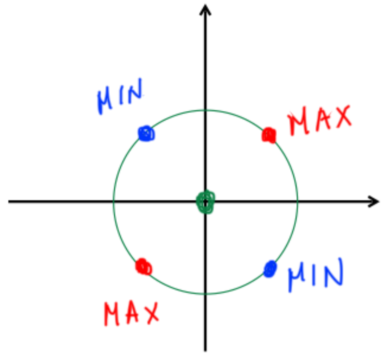
$$L_\lambda(x,y,\lambda) = -x^2 - y^2 + 1 = 0$$

$\boxed{y = x}$ $\quad -2x^2+1=0 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ in $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\boxed{y = -x}$ $\quad -2x^2+1=0 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ in $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

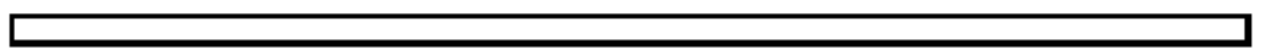


$f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = f\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = \frac{1}{2} \Rightarrow$ MAX

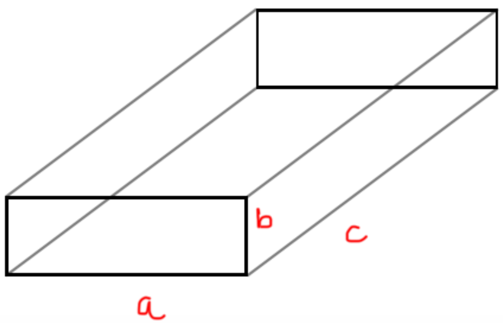
$f\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = f\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = -\frac{1}{2} \Rightarrow$ MIN

(v $(0,0)$ je sedlo)

5.



ℓ metrov $\rightarrow$ 12 palic $\rightarrow$ kvadar $\rightarrow$ $4a$, $4b$, $4c$



$P = 2ab + 2ac + 2bc \longrightarrow$ max

POGOJ: $4a + 4b + 4c = \ell$

Iščemo vezani ekstrem.

$$L(a,b,c,\lambda) = 2ab + 2ac + 2bc - \lambda(4a + 4b + 4c - \ell)$$

$$L_a(a,b,c,\lambda) = 2b + 2c - 4\lambda = 2(b + c - 2\lambda) = 0$$

$$L_b(a,b,c,\lambda) = 2a + 2c - 4\lambda = 2(a + c - 2\lambda) = 0$$

$$L_c(a,b,c,\lambda) = 2a + 2b - 4\lambda = 2(a + b - 2\lambda) = 0$$

$$L_\lambda(a,b,c,\lambda) = -4a - 4b - 4c + \ell = 0$$

$$b + c - 2\lambda = 0 \Rightarrow b = 2\lambda - c$$

$$\parallel$$

$$a + c - 2\lambda = 0 \Rightarrow a = 2\lambda - c$$

$$a + b - 2\lambda = 0 \Rightarrow 2a = 2\lambda \Rightarrow \boxed{\begin{array}{l} a = \lambda \\ b = \lambda \\ c = \lambda \end{array}}$$

$$a = b = c$$

KOCKA

$$a + b + c = \frac{\ell}{4}$$

$$a + a + a = \frac{\ell}{4}$$

$$\boxed{a = \frac{\ell}{12}}$$

12 enakih delov