VAJE 9

OMA VAJE 10

8) b) $f(x) = x \cdot \log x$

$\mathcal{D}_f = (0, \infty)$

$\mathcal{Z}_f = [-1/e, \infty)$

NIČLE: $\cancel{x=0}$, $x = 1$

ZNAK: $x \in (0,1)$ $f(x) < 0$

$x \in (1, \infty)$ $f(x) > 0$

SODOST, LIHOST:

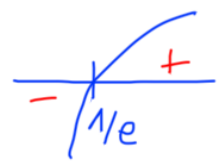
$f(-x) = -x \cdot \log(-x) \neq f(x)$
$\neq -f(x)$

niti liha, niti soda.

$$\lim_{x\to 0^+} x \cdot \log x = \lim_{x\to 0^+} \frac{\log x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} (-x) = 0$$

$$\lim_{x\to \infty} x \log x = \infty$$

$f'(x) = 1 \cdot \log x + x \cdot \frac{1}{x} = \log x + 1 = 0 \Rightarrow x = e^{-1}$ stac. t.



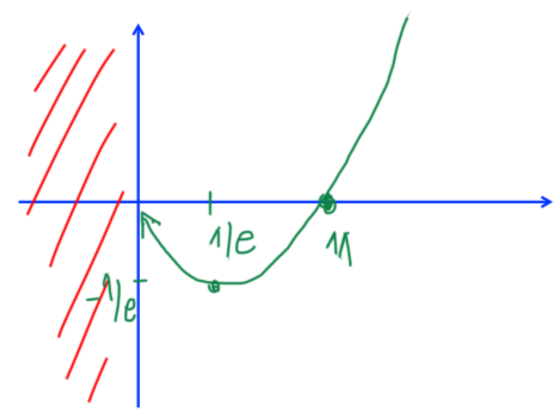
$x \in (0, 1/e)$ $f'(x) < 0$ $f(x) \downarrow$

$x \in (1/e, \infty)$ $f'(x) > 0$ $f(x) \uparrow$

$f''(x) = \frac{1}{x} > 0$, $\forall x \in \mathcal{D}_f \Rightarrow f(x) \cup$ povsod

$f''(e^{-1}) = e > 0$

$\Rightarrow \text{MIN}\left(\frac{1}{e}, -\frac{1}{e}\right)$



(1) $\sqrt{\sin(-0.05)+0.95}$

- F-JA ENE SPREMENLJIVKE: $\boxed{f(x)=\sqrt{\sin(x)+1+x}}$ $\quad f(-0.05)=?$

$x_0=0 \qquad f(x)\approx f(x_0)+f'(x_0)(x-x_0) = f(0)+f'(0)\cdot x$

$f(0)=\sqrt{\sin(0)+1} = \sqrt{1} = 1$

$f'(x) = \dfrac{1}{2\sqrt{\sin(x)+1+x}}\cdot(\cos(x)+1)$

$f'(0) = \dfrac{\cos(0)+1}{2\sqrt{\sin(0)+1}} = \dfrac{2}{2} = 1$

$\Rightarrow \boxed{f(x)\approx 1+x}$

$f(-0.05)\approx 1-0.05 = 0.95$

- F-JA DVEH SPREMENLJIVK: $\boxed{f(x,y)=\sqrt{\sin(x)+y}}$ $\quad f(-0.05,0.95)=?$

$x_0=0,\ y_0=1$

$f(x,y)\approx f(x_0,y_0)+f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)$

$f(x,y)\approx f(0,1)+f_x(0,1)\cdot x+f_y(0,1)(y-1)$

$f(0,1)=\sqrt{\sin(0)+1} = 1$

$f_x(x,y)=\dfrac{\cos(x)}{2\sqrt{\sin(x)+y}} \qquad f_y(x,y)=\dfrac{1}{2\sqrt{\sin(x)+y}}$

$f_x(0,1) = \frac{1}{2}$ $\qquad$ $f_y(0,1) = \frac{1}{2}$

$$\Rightarrow f(x,y) \approx 1 + \frac{1}{2}x + \frac{1}{2}(y-1) = \boxed{\frac{1}{2}(1+x+y)}$$

$$f(-0.05, 0.95) \approx \frac{1}{2}(1-0.05+0.95) = \frac{1}{2}\cdot 2 \cdot 0.95 = 0.95$$

(3) $f(x) = \cos(x)$

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \dots$$

$x_0 = 0$

$f'(x) = -\sin(x)$ $\qquad$ $f(0) = 1$

$f''(x) = -\cos(x)$ $\qquad$ $f'(0) = 0$

$f'''(x) = \sin(x)$ $\qquad$ $f''(0) = -1$

$f^{IV}(x) = \cos(x)$ $\qquad$ $f'''(0) = 0$

$\vdots$ $\qquad$ $f^{IV}(0) = 1$

$\qquad$ $\vdots$

$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$f(x) \approx 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots$$

$$\cos(0.1) \approx 1 - \frac{0.1^2}{2} + \underbrace{\frac{0.1^4}{24}}_{0.0000041} - \underbrace{\frac{0.1^6}{720}}_{0.0000000013} + \dots$$

$$\cos(0.1) \approx 1 - \frac{0.01}{2}$$

$$\cos(0.1) \approx 0.995$$

5. a) $f(x,y)=\dfrac{1}{2x^2+y^2}$ $\qquad \nabla f(x,y) = (f_x(x,y),\ f_y(x,y)) = ?$

$$f_x(x,y) = -(2x^2+y^2)^{-2}\cdot 4x = \frac{-4x}{(2x^2+y^2)^2}$$

$$f_y(x,y) = -(2x^2+y^2)^{-2}\cdot 2y = \frac{-2y}{(2x^2+y^2)^2}$$

$$\nabla f(x,y) = \left(\frac{-4x}{(2x^2+y^2)^2},\ \frac{-2y}{(2x^2+y^2)^2}\right)$$

b) $f(x) = (2x-y)\cdot e^{x+y}$ $\qquad \nabla f(x,y) = ?$

$$f_x(x,y) = 2\cdot e^{x+y} + (2x-y)\cdot e^{x+y}\cdot 1 = (2+2x-y)\cdot e^{x+y}$$

$$f_y(x,y) = -e^{x+y} + (2x-y)\cdot e^{x+y}\cdot 1 = (-1+2x-y)e^{x+y}$$

$$\nabla f(x,y) = \left((2+2x-y)\cdot e^{x+y},\ (-1+2x-y)\cdot e^{x+y}\right)$$

6. $f(x,y) = x^2 + \dfrac{y^2}{4}$ NIVOJNICE, $\mathcal{D}f$ – smo že rešili (glej vaje 9. teden, naloga 1.)

7. $f(x,y) = x^2 y$

a) $f_x(x,y) = 2xy$ $\qquad f_y(x,y) = x^2$

$f_x(1,1) = 2$ $\qquad f_y(1,1) = 1$

$\nabla f(x,y) = (2xy,\ x^2)$

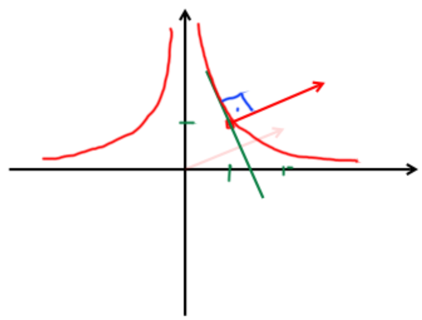
$\nabla f(1,1) = (2,1)$

b) $x^2\cdot y = a$

$y = \dfrac{a}{x^2}$

ČEZ $(1,1) \Rightarrow 1 = \dfrac{a}{1}$

$\boxed{a=1}$



smerni koef. gradienta : $k_G = \frac{1-0}{2-0} = \frac{1}{2}$

$k_G \cdot k_t = \frac{1}{2} \cdot (-2) = -1$

smerni koef. tangente na krivuljo : $y' = -\frac{2}{x^3}$, za $x=1$ $k_t = -2$

PRAVOKOTNI PREMICI

$\Rightarrow$ gradient je v $(1,1)$ pravokoten na krivuljo (nivojnico).

8. $f(x,y) = y(x^2-1)$

NIVOJNICE : $f(x,y) = a$

$y \cdot (x^2-1) = a$

$y = \frac{a}{x^2-1}$

če $\boxed{a=0}$ $\Rightarrow$ premice :

$x = 1$

$x = -1$

$y = 0$

NARIŠEMO ZA : $\boxed{a \neq 0}$

$\boxed{a=1} \Rightarrow \boxed{y = \frac{1}{x^2-1}}$

$\boxed{a=-1} \Rightarrow \boxed{y = \frac{-1}{x^2-1}}$