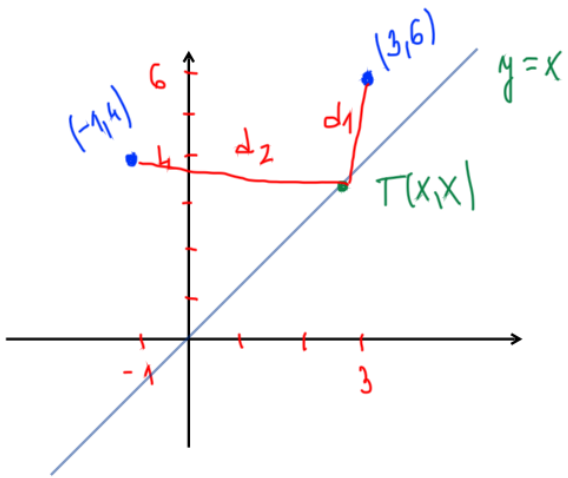
OMA 9. VAJE

2.



$$d_1^2 + d_2^2 \longrightarrow \min$$

$$d_1^2 = (x-3)^2 + (x-6)^2$$

$$d_2^2 = (x+1)^2 + (x-4)^2$$

$$D(x) = d_1^2 + d_2^2 = (x-3)^2 + (x-6)^2 + (x+1)^2 + (x-4)^2$$

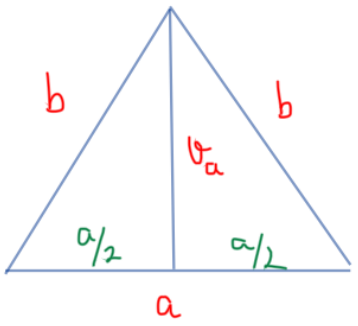
$$D'(x) = 2(x-3) + 2(x-6) + 2(x+1) + 2(x-4) = 8x - 24 = 0$$

$$x = 3$$

$$D''(x) = 8 > 0 \quad \text{za vsako } x$$

$$\Rightarrow T(3,3)$$

3.



ZNAN OBSEG $\quad o = a + 2b \quad \Rightarrow \quad b = \frac{o-a}{2}$

$$p(a) = \frac{a \cdot v_a}{2} = \frac{a \cdot \sqrt{b^2 - \frac{a^2}{4}}}{2} = \frac{a\sqrt{\frac{o^2-2oa+a^2}{4} - \frac{a^2}{4}}}{2} = \frac{a\sqrt{o^2-2oa}}{4}$$

$$p'(a) = \frac{1}{4}\left(\sqrt{o^2-2oa} + a \cdot \frac{1}{2\sqrt{o^2-2oa}} \cdot (-2o)\right) = \frac{1}{4} \cdot \frac{o^2-2oa-oa}{\sqrt{o^2-2oa}}$$

$$p'(a) = \frac{o(o-3a)}{4\sqrt{o^2-2oa}} = 0 \quad \Rightarrow \quad o = 0 \quad /$$

ali $\quad o - 3a = 0$

$o = 3a$ (enakostraničen $\triangle$)

4.

a) $f(x) = x^x \quad / \log$

$\log f(x) = x \cdot \log x \quad /'$

$\frac{1}{f(x)} \cdot f'(x) = 1 \cdot \log x + x \cdot \frac{1}{x}$

$f'(x) = f(x) \cdot (\log x + 1)$

$f'(x) = x^x (\log x + 1)$

b) $f(x) = x^{1/x} \quad / \log$

$\log f(x) = \frac{1}{x} \log x \quad /'$

$\frac{1}{f(x)} \cdot f'(x) = -\frac{1}{x^2} \log x + \frac{1}{x} \cdot \frac{1}{x}$

$f'(x) = f(x) \cdot \frac{1}{x^2}(1 - \log x)$

$f'(x) = x^{1/x} \cdot \frac{1}{x^2}(1 - \log x) = x^{\frac{1}{x}-2}(1 - \log x)$

(5) Lokalni ekstremi funkcije

$g(x) = 12x^5 + 15x^4 - 40x^3 + 5$

Stacionarne točke

$g'(x) = 60x^4 + 60x^3 - 120x^2 = 60x^2(x^2 + x - 2) = 60x^2(x+2)(x-1) = 0 \Rightarrow x_{1,2} = 0 \quad x_3 = -2 \quad x_4 = 1$

$g''(x) = 240x^3 + 180x^2 - 240x = 60x(4x^2 + 3x - 4)$

$g''(-2) < 0 \Rightarrow$ LOK. MAX $(-2, 181)$

$g''(1) > 0 \Rightarrow$ LOK. MIN $(1, -8)$

$g'''(x) = 720x^2 + 360x - 240$

$g''(0) = 0$ in $g'''(0) \neq 0 \Rightarrow (0,5)$ prevoj

(6.) L'Hospitalovo pravilo

a) $\lim\limits_{x \to 0} \frac{x + \sin(2x)}{x - \sin(3x)} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x \to 0} \frac{1 + 2\cos(2x)}{1 - 3\cos(3x)} = \frac{1+2}{1-3} = -\frac{3}{2}$

b) $\lim\limits_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x = \lim\limits_{x \to \pi/2} \frac{\left(\frac{\pi}{2} - x\right)\sin(x)}{\cos(x)} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x \to \pi/2} \frac{-\sin(x) + \left(\frac{\pi}{2} - x\right)\cos(x)}{-\sin(x)} = \frac{-1+0}{-1} = 1$

c) $\lim_{x\to\infty} \frac{x^3 - x^2 + e^x}{2x^3} \overset{"\frac{\infty}{\infty}"}{=} \lim_{x\to\infty} \frac{3x^2 - 2x + e^x}{6x^2} \overset{"\frac{\infty}{\infty}"}{=} \lim_{x\to\infty} \frac{6x - 2 + e^x}{12x} \overset{"\frac{\infty}{\infty}"}{=} \lim_{x\to\infty} \frac{6 + e^x}{12} = \infty$

d) $\lim_{x\to 0} \frac{1 - \cos(x)}{x^2} \overset{"\frac{0}{0}"}{=} \lim_{x\to 0} \frac{\sin(x)}{2x} \overset{"\frac{0}{0}"}{=} \lim_{x\to 0} \frac{\cos(x)}{2} = \frac{1}{2}$

e) $\lim_{x\to-\infty} xe^x = \lim_{x\to-\infty} \frac{x}{\frac{1}{e^x}} \overset{"\frac{\infty}{\infty}"}{=} \lim_{x\to-\infty} \frac{1}{-\frac{1}{e^x}} = \lim_{x\to-\infty} -e^x = 0$