Diskretne strukture UNI, 18. 11. 2021 (11.15 - 13.00, P18)

1. Katere izmed spodnjih formul so enakovredne?

$$\begin{aligned} A &= \exists x(\forall y P(x,y) \Rightarrow \forall y R(x,y)), \\ B &= \exists x(\forall y P(y,x) \Rightarrow \forall y R(x,y)), \\ C &= \exists x(\forall y P(x,y) \Rightarrow \forall y R(y,x)). \end{aligned}$$

Poskusimo A zapisati v prenexni obliki:

$$A \sim \exists x\,(\neg \forall y\, P(x,y) \vee \forall y\, R(x,y)) \sim \exists x \forall y\,(\neg\forall y\, P(x,y) \vee R(x,y)) \sim$$

$W \vee \forall x P(x) \sim \forall x (W \vee P(x))$ $\qquad$ $\neg \forall x P(x) \sim \exists x \neg P(x)$

$$\sim \exists x \forall y\,(\exists y \neg P(x,y) \vee R(x,y)) \sim \exists x \forall y\,(\exists z \neg P(x,z) \vee R(x,y)) \sim$$

preimenujemo y v z

$$\sim \exists x \forall y \exists z\,(\neg P(x,z) \vee R(x,y)).$$

Mogoče bo formule lažje primerjati, če $\exists x$ (na začetku) "nesemo v oklepaje"...

$$A \sim \exists x\,(\exists y \neg P(x,y) \vee \forall y\, R(x,y)) \sim \exists x \exists y \neg P(x,y) \vee \exists x \forall y\, R(x,y) \sim (*)$$

$\exists x (P(x) \vee R(x)) \sim \exists x P(x) \vee \exists x R(x)$

Podobno:

$B \sim \ldots \ldots \sim \exists x \exists y \neg P(y,x) \vee \exists x \forall y\, R(x,y)$

$C \sim \ldots \ldots \sim \exists x \exists y \neg P(x,y) \vee \exists x \forall y\, R(y,x)$

$$(*) \sim \exists y \exists x \neg P(x,y) \vee \exists x \forall y\, R(x,y) \sim \exists x \exists y \neg P(y,x) \vee \exists x \forall y\, R(x,y)$$

=

zamenjamo vlogi x in y

Torej $A \sim B$.

Sumimo, da $A \not\sim C$... Izberimo $P(x,y) = 1$ za vse x, y iz pod. pog.

Tedaj je $A \sim \exists x \forall y\, R(x,y)$ in $C \sim \exists x \forall y\, R(y,x)$.

pod. pogovora ..... naravna števila, v tem primeru $A \sim 0$ in $C \sim 1$

$R(x,y)$ ...... $x \geq y$

Torej A in C res nista enakovredni.

3. Dane so množice $A = \{1,2,3\}$, $B = \{2,3,4\}$ in $C = \{0,1,4,5\}$. Določi naslednje množice:

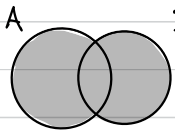
(a) $C + (A \cup C)$,

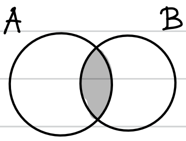
(b) $(B \setminus A) \cap C$,

(c) $\mathcal{P}(A \cap B) \setminus C$,

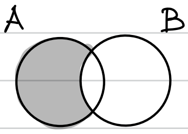
(d) $\mathcal{P}(A \cap C) + \mathcal{P}(B \cap C)$.

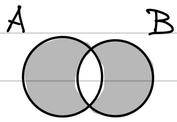
Operacije na množicah:





$A \cup B = \{x : x \in A \lor x \in B\}$ $\qquad$ $A \cap B = \{x : x \in A \land x \in B\}$





$A \setminus B = \{x : x \in A \land \neg(x \in B)\}$ $\qquad$ $A + B = \{x : x \in A \veebar x \in B\}$

(a) $C + (A \cup C) = \{0,1,4,5\} + \{0,1,2,3,4,5\} = \{2,3\}$.

(b) $(B \setminus A) \cap C = \{4\} \cap C = \{4\}$.

(c) $\mathcal{P}(A \cap B) \setminus C = \mathcal{P}(\{2,3\}) \setminus C =$ $\qquad$ $\mathcal{P}(A) = \{X : X \subseteq A\}$

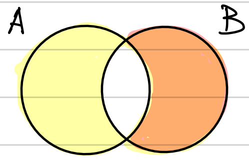
$= \{\underbrace{\{\}}_{\emptyset}, \{2\}, \{3\}, \{2,3\}\} \setminus \{0,1,4,5\} = \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$.

(d) $\mathcal{P}(A \cap C) + \mathcal{P}(B \cap C) = \mathcal{P}(\{1\}) + \mathcal{P}(\{4\}) = \{\{1\}, \{4\}\}$.

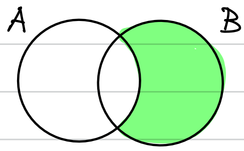
5. Ali velja

(a) $(A+B)\setminus A = B\setminus A$,

leva stran



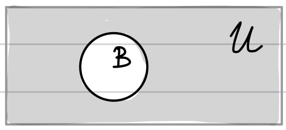
desna stran



Izgleda enako.... preverimo, da je res enako z uporabo osnovnih enakosti z množicami.

$$\underline{(A+B)\setminus A} = ((A\setminus B)\cup(B\setminus A))\setminus A = (A\cap B^c \cup B\cap A^c)\cap A^c =$$

$A+B=(A\setminus B)\cup(B\setminus A)$ $\quad$ $A\setminus B = A\cap B^c$



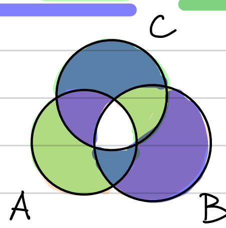
$$= A\cap B^c\cap A^c \cup B\cap A^c\cap A^c = B^c\cap\emptyset \cup B\cap A^c =$$

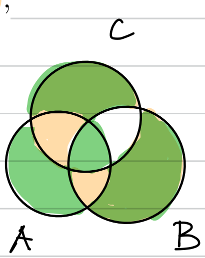
$(A\cup B)\cap C = A\cap C\cup B\cap C$ $\quad$ $A\cap A^c=\emptyset$, $A\cap A = A$

$$= B\cap A^c = \underline{B\setminus A},$$ torej sta res enaki.

$A\cap\emptyset=\emptyset$, $\emptyset\cup A = A$

(b) $(A+B)+(A+C) = A+(B+C)$,





Ne izgleda enako....

Poiščimo množice A, B in C, da utemeljimo, da res ni enako.

Izberimo $A=B=C=\{1\}$, tedaj:

$$(A+B)+(A+C) = \emptyset+\emptyset=\emptyset \quad \text{in} \quad A+(B+C) = \{1\}+\emptyset = \{1\},$$

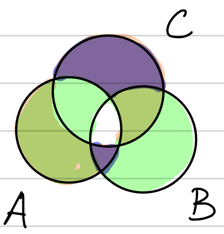
res nista enaki (v splošnem).

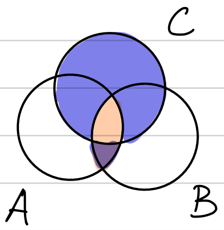
Še en protiprimer: $A=C=\{1\}$, $B=\emptyset$, tedaj:

$$\underbrace{(A+B)}_{\{1\}}+\underbrace{(A+C)}_{\emptyset} = \{1\} \quad \text{in} \quad A+\underbrace{(B+C)}_{\{1\}} = \emptyset \ldots$$

(f) $(A+C)\setminus(A+B) \subseteq (A\cap B)+C$,

Skicirajmo oba Vennova diagrama:





Izgleda, da vsebovanost velja...

$$(A+C)\setminus(A+B) = ((A\setminus C)\cup(C\setminus A))\setminus((A\setminus B)\cup(B\setminus A)) =$$

$$= (A\cap C^c \cup C\cap A^c)\cap(A\cap B^c\cup B\cap A^c)^c =$$

$$= (A\cap C^c\cup C\cap A^c)\cap((A^c\cup B^{cc})\cap(B^c\cup A^{cc})) =$$

$(A\cup B)^c = A^c\cap B^c$

$(A\cap B)^c = A^c\cup B^c$

$$= (A\cap C^c\cup C\cap A^c)\cap(\underbrace{(A^c\cup B)\cap(B^c\cup A)}_{(A^c\cap B^c \cup \underbrace{\cancel{A^c\cap A}}_{=\emptyset} \cup \underbrace{\cancel{B\cap B^c}}_{=\emptyset}\cup B\cap A)}) =$$

$A^{cc} = A$

$$= \underbrace{\cancel{A\cap C^c\cap A^c\cap B^c}}_{\emptyset} \cup A\cap C^c\cap B\cap A \cup C\cap A^c\cap A^c\cap B^c \cup \underbrace{\cancel{C\cap A^c\cap B\cap A}}_{=\emptyset} =$$

$$= A\cap B\cap C^c \cup A^c\cap B^c\cap C.$$

$$(A\cap B)+C = A\cap B\cap C^c \cup C\cap(A\cap B)^c = A\cap B\cap C^c\cup C\cap(A^c\cup B^c) =$$

$$= A\cap B\cap C^c\cup C\cap A^c\cup C\cap B^c =$$

$$= A\cap B\cap C^c\cup C\cap A^c\cup C\cap B^c\cap(A\cup A^c) =$$

$A\cap \mathcal{U} = A$

$A\cup A^c = \mathcal{U}$

$$= A\cap B\cap C^c\cup C\cap A^c\cup A\cap B^c\cap C\cup A^c\cap B^c\cap C$$

Torej vsebovanost res velja.