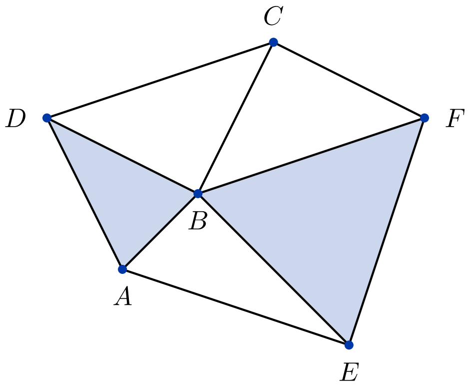
Computational topology
Lab work, 6^{th} week

1. Find the open stars $\mathrm{st}(A)$, $\mathrm{st}(AB)$ and the links $\mathrm{lk}(A)$, $\mathrm{lk}(AB)$ for the simplicial complex given below.



2. The simplicial complex K contains the following simplices:

$$\langle v_0\rangle, \langle v_1\rangle, \langle v_2\rangle, \langle v_3\rangle, \langle v_4\rangle, \langle v_0, v_1\rangle, \langle v_0, v_3\rangle, \langle v_1, v_3\rangle, \langle v_0, v_1, v_2\rangle.$$

   (a) Add any simplices that are missing from K.

   (b) Draw the Hasse diagram of K.

   (c) Find the open stars $\mathrm{st}(\langle v_1\rangle)$, $\mathrm{st}(\langle v_1, v_3\rangle)$ and the links $\mathrm{lk}(\langle v_2\rangle)$, $\mathrm{lk}(\langle v_0, v_3\rangle)$. Mark them on the Hasse diagram as well.

3. For each of the following triangulations determine if it is a triangulation of a surface.

   A: `[(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)]`

   B: `[(1, 2, 3), (1, 2, 4), (2, 3, 5), (2, 3, 6), (3, 5, 7)]`

   C: `[(1, 2, 3), (2, 3, 4), (3, 4, 5),`
   `(4, 5, 6), (1, 5, 6), (1, 2, 6)]`

   D: `[(1, 2, 4), (2, 4, 6), (2, 3, 6), (3, 6, 8), (1, 3, 8),`
   `(1, 4, 8), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9),`
   `(4, 8, 9), (4, 5, 9), (1, 5, 7), (1, 2, 7), (2, 7, 9),`
   `(2, 3, 9), (3, 5, 9), (1, 3, 5)]`

   E: `[(1, 2, 4), (2, 4, 6), (2, 3, 6), (3, 6, 8), (1, 3, 8),`
   `(1, 5, 8), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9),`
   `(5, 8, 9), (4, 5, 9), (1, 5, 7), (1, 2, 7), (2, 7, 9),`
   `(2, 3, 9), (3, 4, 9), (1, 3, 4)]`

   F: `[(1, 2, 3), (1, 3, 4), (2, 3, 4), (4, 5, 6)]`

   G: `[(1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (2, 5, 6), (1, 2, 6)]`

   H: `[(1, 3, 5), (1, 2, 6), (1, 5, 6), (1, 2, 4), (1, 3, 4),`
   `(2, 3, 5), (2, 3, 6), (2, 4, 5), (3, 4, 6), (4, 5, 6)]`

   (a) Find the Euler characteristics for all of these simplicial complexes.

   (b) For each case check if the given triangulation belongs to a surface (a 2-dimensional triangulated manifold).

   (c) Find the number of boundary components for all of the surfaces.

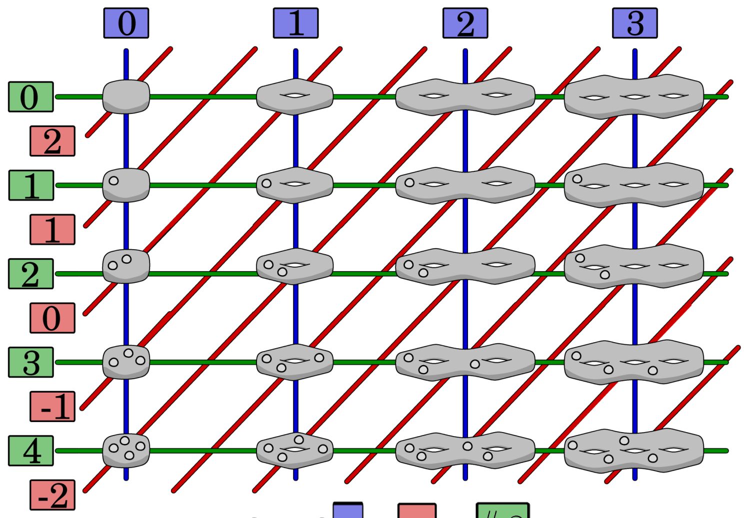
(d) For each of the surfaces determine if it is orientable or not.

(e) Determine the genus of each orientable surface and the genus of non-orientable surfaces with no boundary.

(f) Name each of the surfaces.

Use the following array to keep track of the results.

	Euler characteristic	manifold Y/N	# of boundary components	orientable Y/N	genus	name
A						
B						
C						
D						
E						
F						
G						
H						



$$2 = 2\boxed{g} + \boxed{\chi} + \boxed{\#\partial}$$

genus	0	1	2
orientable	S^2	T	$T\#T$
non-orientable		P	$P\#P$

4. Let $S = \{A(0,0), B(0,1), C(0.5,0.5), D(1,2), E(1.5,1.5), F(2,0), G(2,1.5), H(2.5,1)\} \subset \mathbb{R}^2$. Build the Vietoris-Rips complex $\text{Rips}(S,R)$ for

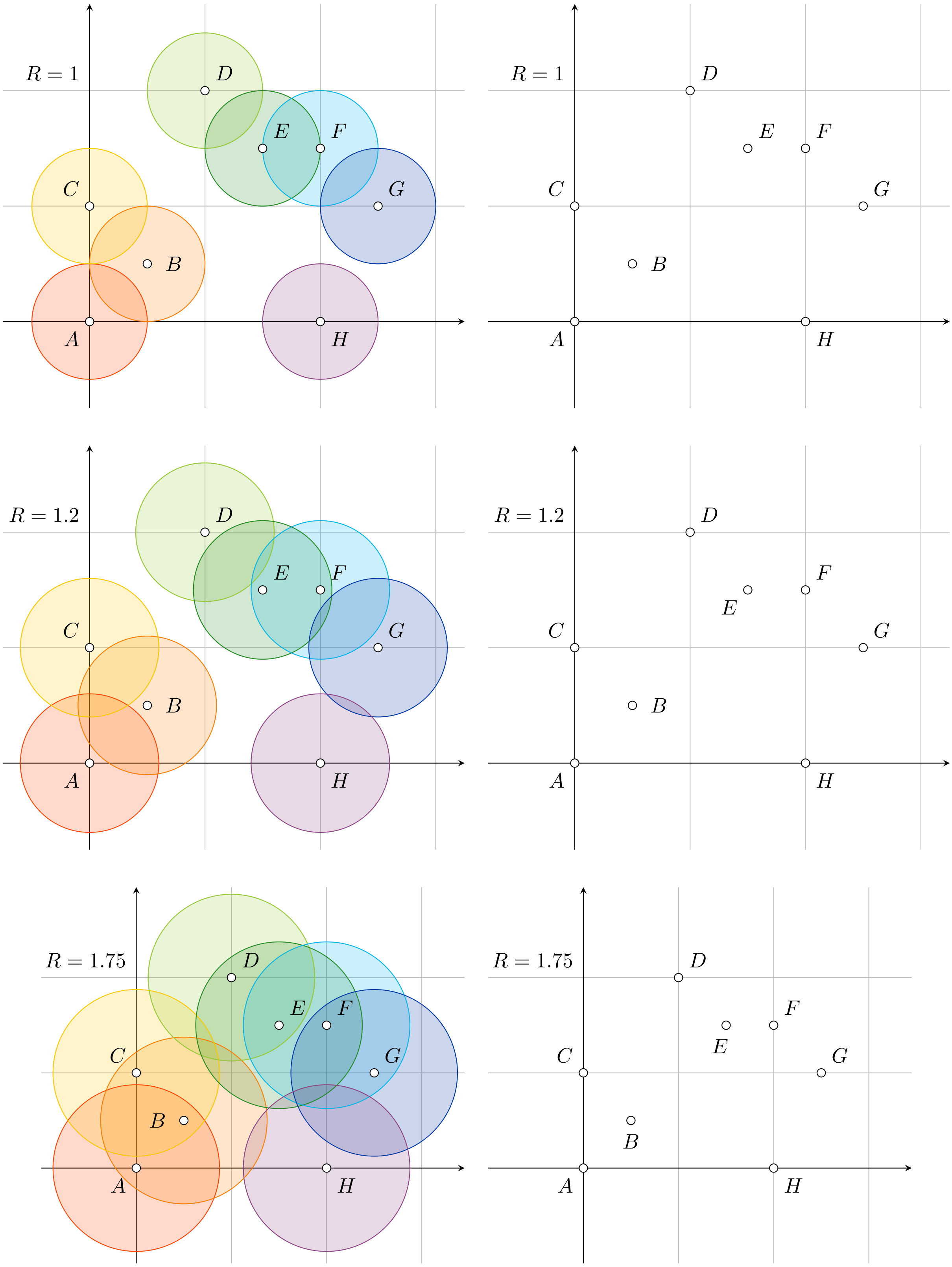
   (a) $R = 1$,

   (b) $R = 1.2$,

   (c) $R = 1.75$.

   In each case list all the simplices and determine its dimension.

   Assuming there is a sensor placed at each point of S and all sensors can detect points that are at distance 1.75 or less, is the area covered by the sensors connected? Does it contain any holes?



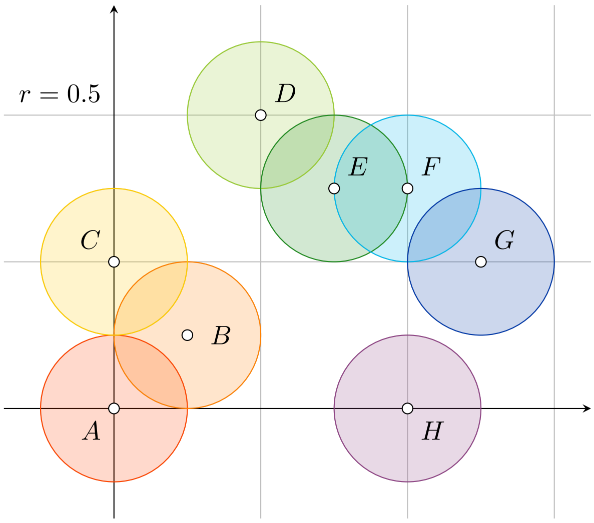
5. Let $S = \{A(0,0), B(0,1), C(0.5,0.5), D(1,2), E(1.5,1.5), F(2,0), G(2,1.5), H(2.5,1)\} \subset \mathbb{R}^2$. Build the Čech complex $\mathrm{Cech}(S, r)$ for

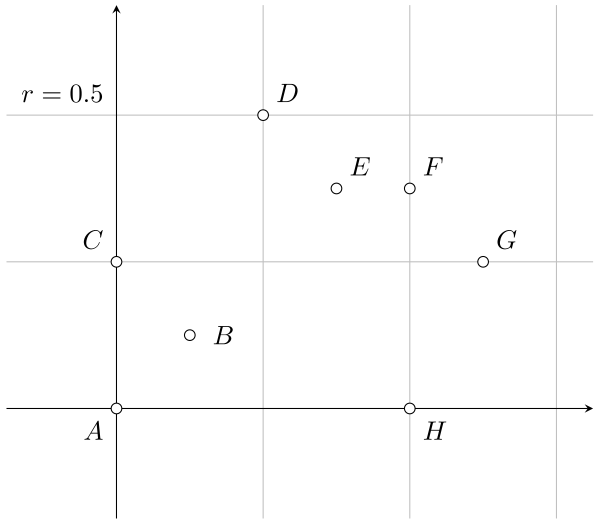
   (a) $r = 0.5$,

   (b) $r = 0.6$,

   (c) $r = 0.875$.

   In each case list all the simplices and determine its dimension.


$r = 0.5$
D
E
F
C
G
B
A
H


$r = 0.5$
D
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G
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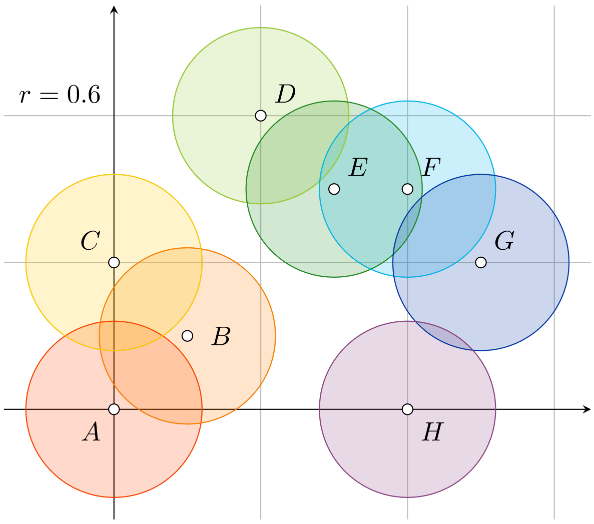
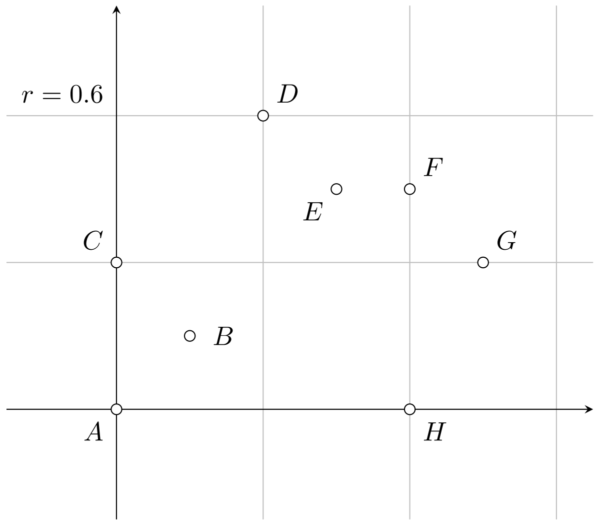
H


$r = 0.6$
D
E
F
C
G
B
A
H


$r = 0.6$
D
F
E
C
G
B
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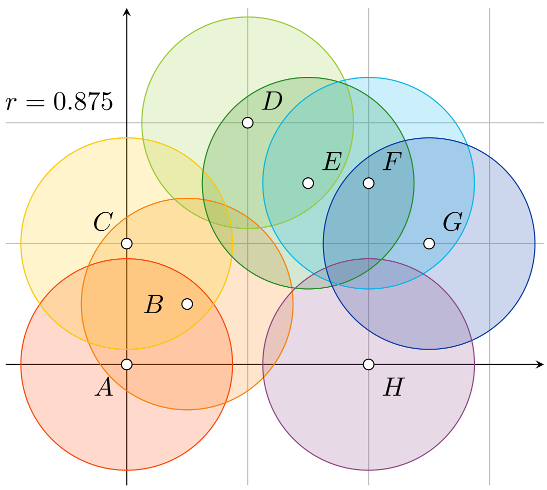
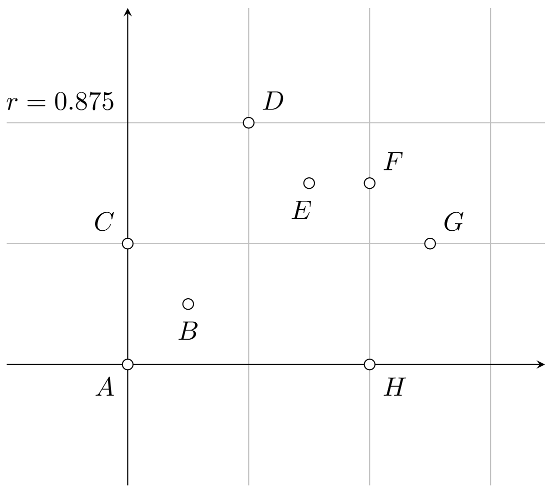
H


$r = 0.875$
D
E
F
C
G
B
A
H


$r = 0.875$
D
F
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C
G
B
A
H