Computational topology

Lab work, 9^{th} week

1. Calculate the following products:

   (a) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and } \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

   (b) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & k \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and } \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & k \end{bmatrix},$$

   (c) $$\begin{bmatrix} 1 & k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and } \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 & k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

   Identify which of these swap two rows/columns, add a multiple of a row/column to another row/column and multiply a row/column with a number.

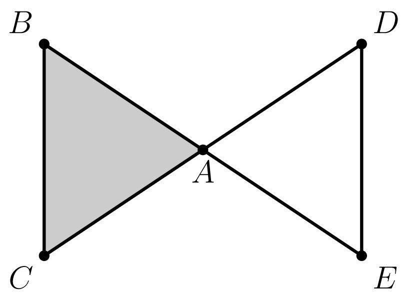
2. For the simplicial complex X in the figure below

   (a) write down the chain groups $\mathcal{C}_2$, $\mathcal{C}_1$ and $\mathcal{C}_0$,

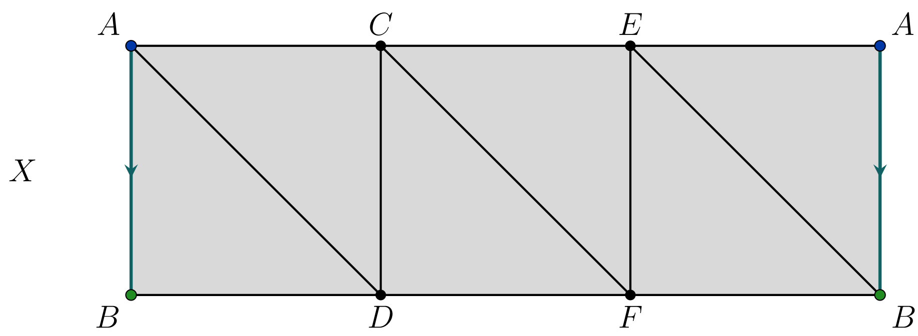
   (b) write down the matrices D_n for the boundary homomorphisms $\partial_n\colon \mathcal{C}_n \to \mathcal{C}_{n-1}$ for $n = 0, 1, 2, 3$,

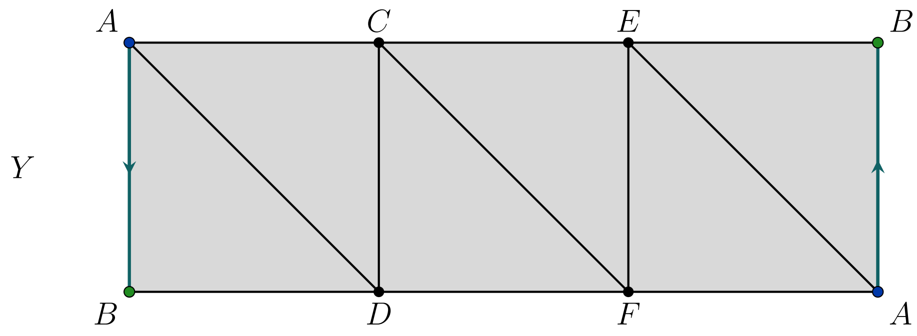
   (c) compute the homology groups,

   (d) collapse the free faces and determine how this changes the boundary matrices.

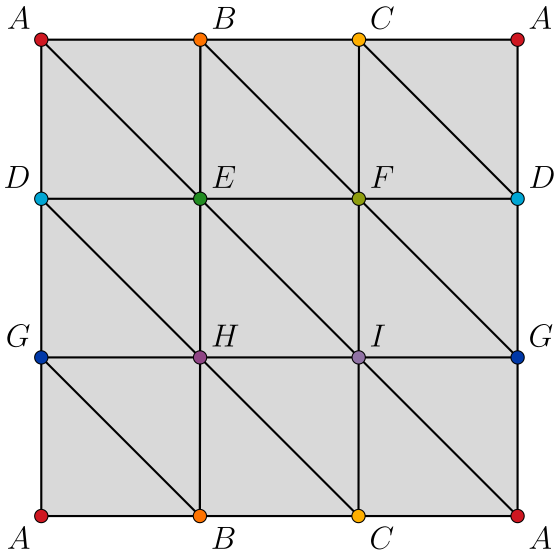


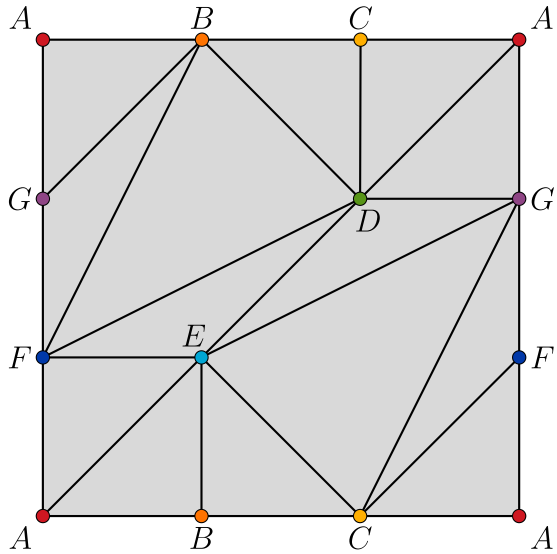
3. Write down the corresponding chain groups $\mathcal{C}_2$, $\mathcal{C}_1$ and $\mathcal{C}_0$ for the Moebius strip (use the triangulation from last week) and the matrices D_n for the boundary homomorphisms $\partial_n\colon \mathcal{C}_n \to \mathcal{C}_{n-1}$. Compute the homology.

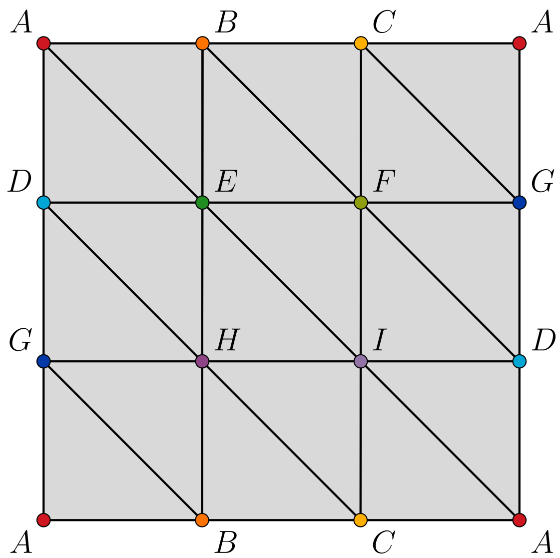


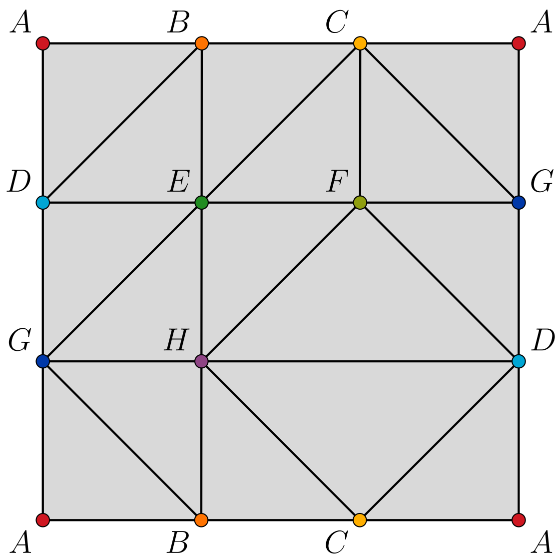


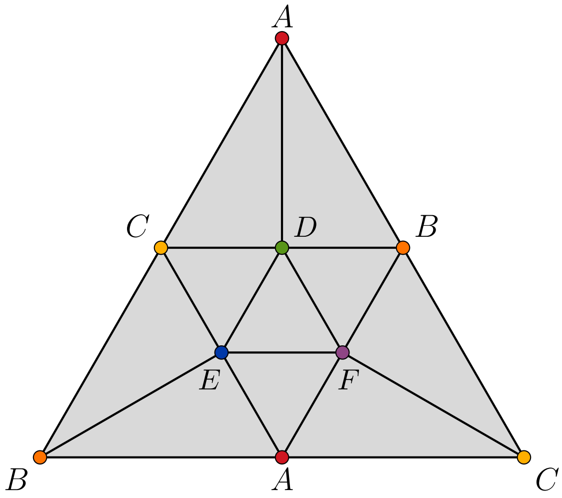
4. Write down the corresponding chain complexes $\mathcal{C}_2$, $\mathcal{C}_1$ and $\mathcal{C}_0$ and the matrices D_2 and D_1 for the boundary homomorphisms $\partial_2\colon \mathcal{C}_2 \to \mathcal{C}_1$ and $\partial_1\colon \mathcal{C}_1 \to \mathcal{C}_0$ for the torus, the Klein bottle and the projective plane. Use your favourite computational topology software to compute the homology groups.







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