

1. It's a sunny April day and Victor is craving for a beer upon returning home. Problem: There is not a single beer can in the fridge. He quickly puts a beer can (which is at 24°C at the moment) in the fridge (which is constantly at 4°C) and waits for half an hour. Once he gets the beer out of the fridge, it has 14°C. (Victor keeps an infrared thermometer always handy at home... )
  - (a) Write down and solve the differential equation, which determines the temperature of the beer can depending on time.  
*Hint:* The cooling rate is proportional to the difference in temperatures.
  - (b) How long should Victor keep the beer in the fridge to cool it down to 9°C?
2. Find the general solution of the differential equation

$$y' = 2x(1 + y^2),$$

and the solution satisfying the condition  $y(1) = 0$ .

3. Find the general solution of the *logistic differential equation*

$$y' = cy \left(1 - \frac{y}{a}\right),$$

and the solution satisfying the condition  $y(0) = b$ .

4. Write an Octave function `[ t, Y ] = euler(f, [ t0, tk ], y0, h)`, which solves the differential equation

$$y' = f(t, y) \quad \text{with initial condition} \quad y(t_0) = y_0$$

using the Euler method with step size  $h$ . The function should return a set of function values  $Y$  evaluated at times  $t$ .

Solve DE's above using this method. Compare exact and numerical solutions.

5. Write an Octave function `[ t, Y ] = rk4(f, [ t0, tk ], y0, h)`, which solves the differential equation

$$y' = f(t, y) \quad \text{with initial condition} \quad y(t_0) = y_0$$

using the classical order 4 Runge–Kutta method; for step size  $h$  define

$$\begin{aligned} k_1 &= hf(t_i, y_i) \\ k_2 &= hf(t_i + h/2, y_i + k_1/2) \\ k_3 &= hf(t_i + h/2, y_i + k_2/2) \\ k_4 &= hf(t_i + h, y_i + k_3) \end{aligned}$$

and evaluate the next value with

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

Solve DE's above using this method. Compare exact and numerical solutions.