

Diskretne strukture UNI

Vaje 6

1. Katere izmed spodnjih formul so enakovredne?

$$\begin{aligned}A &= \exists x(\forall y P(x, y) \Rightarrow \forall y R(x, y)), \\B &= \exists x(\forall y P(y, x) \Rightarrow \forall y R(x, y)), \\C &= \exists x(\forall y P(x, y) \Rightarrow \forall y R(y, x)).\end{aligned}$$

2. Pokaži, da sta formuli

$$F_1 = \neg \exists x((\neg R(x) \Rightarrow P(x)) \wedge (Q(x) \Rightarrow R(x)))$$

in

$$F_2 = \forall x(P(x) \Rightarrow Q(x)) \wedge \neg \exists y R(y)$$

enakovredni.

3. Dane so množice $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ in $C = \{0, 1, 4, 5\}$. Določi naslednje množice:

- (a) $C + (A \cup C)$,
- (b) $(B \setminus A) \cap C$,
- (c) $\mathcal{P}(A \cap B) \setminus C$,
- (d) $\mathcal{P}(A \cap C) + \mathcal{P}(B \cap C)$.

4. Na ravni elementov pokaži, da velja

- (a) $A \subseteq B \Rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$,
- (b) $A \subseteq B \Leftrightarrow A \cap B = A$.

5. Ali velja

- (a) $(A + B) \setminus A = B \setminus A$,
- (b) $(A + B) + (A + C) = A + (B + C)$,
- (c) $(A \setminus B) + (C \setminus B) = (A + C) \setminus B$,
- (d) $(A + C) \setminus (A + B) = (A \cap B) + C$,
- (e) $(A + C) \setminus (A + B) = (A \cap B) + C$ pod pogojem $C \subseteq A \cap B$,
- (f) $(A + C) \setminus (A + B) \subseteq (A \cap B) + C$,
- (g) $(A + B) \setminus C \subseteq (B \setminus (A + C)) \cup (A \setminus (B \cup C))$,
- (h) $(A + B) \setminus C = (B \setminus (A + C)) \cup (A \setminus (B \cup C))$, če sta A in B disjunktni,
- (i) $(A + B) \setminus C = (A \cup C) + (A \cup B)$,
- (j) $(A \cap C) + (B \cap C) = C \setminus (A \cap B)$,
- (k) $(A \cap C) + (B \cap C) \subseteq C \setminus (A \cap B)$,
- (l) $(A \cap C) + (B \cap C) = C \setminus (A \cap B)$, če je $C \subseteq A \cup B$,
- (m) $(A \setminus C) + B = (A + B) \setminus C$,
- (n) $(A \setminus C) + B \subseteq (A + B) \setminus C$,
- (o) $(A \setminus C) + B = (A + B) \setminus C$, če je $C \subseteq A \setminus B$?

6. Pokaži, da množice $B \cap C$, $(B + C) \cap A$ in $(A + C) \setminus B$ predstavljajo razbitje za množico $A \cup C$.